Periodic Research

Comparative Study of Thermal Conductivity of Two-Phase Systems using Resistor Model

Abstract

The theoretical model is presented to develop effective thermal conductivity of two-phase systems for the different values of thermal conductivity of the constituent phases with their volume fractions. In this model, all arrangement has been divided into unit cells, each of which contains a regular three dimensional (3-D) cubic geometry like a cylinder or ellipsoid. We use resistor model, to determine the effective thermal conductivity (ETC). The correction term F, which depends upon porosity, is introduced. We have large number of samples in this theoretical model. So here, we get large number of comparisons with other models and experimental values cited in the given literature.

Keyword: Effective thermal conductivity (ETC); two-phase system; correction term

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Introduction

The theoretical modeling for two-phase systems is a challenging task for engineer and physicists. There are many fields like- the explosive industry, the ceramics industry, nuclear reactors and in missile technology, in which the study of thermal parameters of these two-phase systems is valuable. So the knowledge of thermal conductivity is very essential. Because these parameters are very helpful for engineers, architects, and physicists. The ETC depends on different factors like thermal conductivity, porosity, size of the particles, and packing of the constituent phases. Accounting for all these factors in order to predict ETC is a very complex affair. In various literature one finds several efforts [1-6] in which the situation has been simplified by assuming that the particles are of specific shape and arranged in a particular geometries within the continuous phase.

In the present paper we have tried to fill the space arrangement of cells of equal size with the minimal surface energy and a theoretical model has been proposed to predict ETC of two-phase system with a regular 3-D cubic geometry. We know the resistor model has been applied to determine ETC of the unit cell. Expressions for the porosity with correction factor have been obtained empirically by simulating experimental data present in the various literature. The present model is very simple and provides wider applicability to 3-D cubic geometry. Its ability to predict correctly the ETC of two-phase system. This system having high ratios of thermal conductivities of their constituent phases.

Theory

To solve the problem, we take three assumptions in our model- (i) the contact resistance between the solid and fluid phase is negligible, (ii) the mixture is homogeneous throughout and no transfer of heat occurs by way of convection or radiation and (iii) the heat flows along the x-axis and the flux lines remain parallel during the heat flow.

Let the grains of the solid phase be 3-D cubic geometry principal axes 2a, 2b and 2c (a<c). Suppose these grains be located at the corners of a sample cube of side 2b each. Their distribution in 2-D is shown in figure 1(a). The geometry of a unit cell is shown in figure 1(b).

Assume that the origin of coordinate axes be located at the centre of the 3-D cubic geometry. Consider one slice bounded by two phanes at distances x and x+dx. This section shown in figure 1(c), which is divided into four quadrants. One such section is shown in figure 1(d). this section is further divided by planes perpendicular to the z-axes. This results in the section of rectangular bars, which is shown in figure 1(e). Suppose the length of the bar be b and the cross-sectional area will be dxdz.
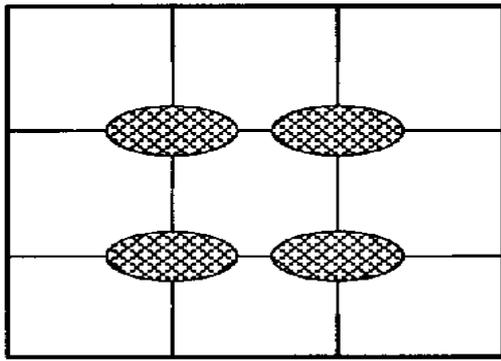
Periodic Research

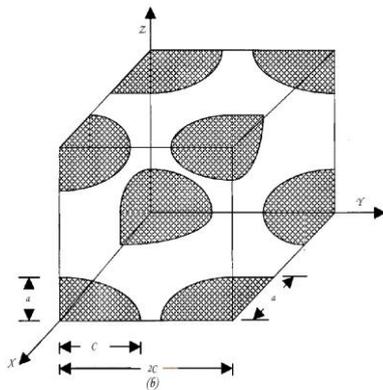

Fig. 1(a) particles distribution in 2-D.

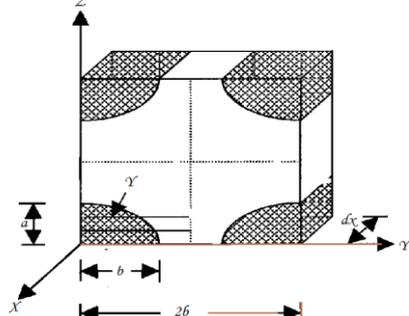Fig. 1(b) geometry of a unit cell.
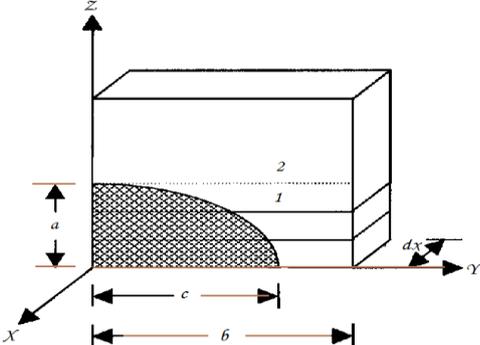

Fig. 1(c) one section of unit cell.
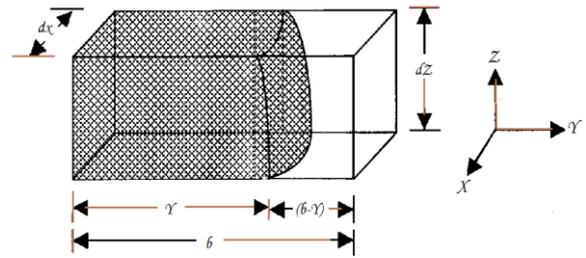

Fig. 1(d) one part of the section

Fig. 1(e) rectangular bar.

The volume fraction of the solid phase will be

$(ydxdz)/(bdx\,dz)= y/b$ 　　　　(1)

Similarly, the volume fraction of the fluid phase will be

$\{(b-y)dxdz\}/(bdxdz)=(1-y/b)$ 　　　(2)

The terms (y/b) and (1-y/b) are equivalent to the 1-D porosity as used by Cheng and Vachon [7]. Considering various components as resistors one can take a combination of such resistors to predict ETC. Because these elements form parallel resistors with respect to the direction of heat flow, so we use the resistor model the thermal conductivity of the bar will be

$\lambda' = \lambda_1 (y/b) + \lambda_2 (1-y/b)$ 　　　(3)

where λ_1 and λ_2 are the thermal conductivities of solid and fluid phases, respectively. Now with the help of reference [8], we do same process for 3-D cubic geometry and at last we get the following expression as below-

$$\lambda_e = \frac{\lambda_2\,[\,(\lambda_1-\lambda_2)\{\sqrt{\pi/6}\}\phi_1^{1/2}+\lambda_2\,]}{[(1-\{\sqrt{\pi/6}\}\phi_1^{1/2})(\lambda_1-\lambda_2)\{\sqrt{\pi/6}\}\phi_1^{1/2}]+\lambda_2} \quad (4)$$

But for practical utilization, we have to modify (4) by incorporating some correction term with reference [9] Tareev has shown that, during the flow of electric flux from one dielectric to another dielectric medium, the deviation of flux lines in any medium depends upon the ratio of the dielectric constants of the two media. Using these phenomena we can have the concentration of thermal flux altered from its previous value as it passes through another medium and that the amount is a function of the thermal conductivities of the constituent phases. So a zigzag path of flux lines in the bulk and also alters the density of flux lines in the constituent phases. Therefore, (4) can be written as-

$$\lambda_e = \frac{\lambda_2\,[\,(\lambda_1-\lambda_2)\{\sqrt{\pi/6}\}F^{1/2}+\lambda_2\,]}{[(1-\{\sqrt{\pi/6}\}F^{1/2})(\lambda_1-\lambda_2)\{\sqrt{\pi/6}\}F^{1/2}]+\lambda_2} \quad (5)$$

Rearranging (5) we get

$AF + BF^{1/2} + C =$ 　　　　(6)

Where $A = [\lambda_e\,(\lambda_1-\lambda_2)]$, $B = [\{\sqrt{(\pi/6)}\}\,(\lambda_1-\lambda_2)\,(\lambda_2-\lambda_e)]$ and $C = \lambda_2\,(\lambda_2-\lambda_e)$.

Results and Discussion

Tables 1 and 2 cite experimental results of ETC and other data reported in the literature. Without using correction term, from equation (4), we have large deviation from the experimental results. This prompted the introduction of a correction in porosity. The correction term introduced for each sample has been computed using (6) and plotted with $\phi_1^{2/3}\exp(\frac{\lambda_2}{\lambda_1+\lambda_2})$._____Such plots of $\phi_1^{2/3}\exp(\frac{\lambda_2}{\lambda_1+\lambda_2})$ versus $F^{1/2}$ are shown in figure 2 and 3. It is observed from the figure that $F^{1/2}$

Periodic Research

increases roughly linearly with increasing $\phi_1^{2/3} \exp(\frac{\lambda_2}{\lambda_1 + \lambda_2})$. We have used the curve fitting technique and found that the expression

$$F.^{1/2} = C_1 \; \phi_1^{2/3} \exp\left(\frac{\lambda_2}{\lambda_1 + \lambda_2}\right) + C_2 \qquad (7)$$

best fit the curve obtained in figure 2 and 3 where C_1 and C_2 are constants. These constants are different for different type of materials. The values of these constants for solid-air, emulsion, suspension, granular and solid-solid two-phase systems are 0.82641 and 0.063, for Al-air system constants are 0.04113 and 0.71273, respectively.
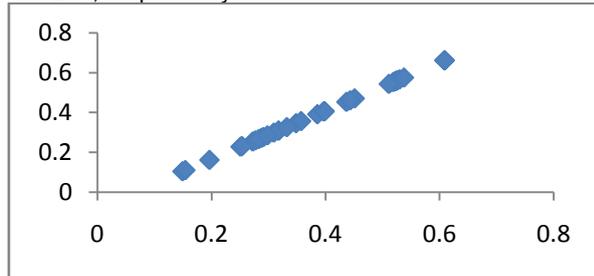
Fig. 2 the variation of porosity correction term $F.^{1/2}$ vs $\phi_1^{2/3} \exp(\frac{\lambda_2}{\lambda_1 + \lambda_2})$.
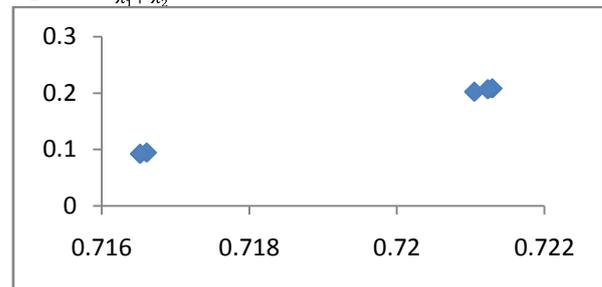
Fig. 3 the variation of porosity correction term $F.^{1/2}$ vs $\phi_1^{2/3} \exp(\frac{\lambda_2}{\lambda_1 + \lambda_2})$.

Tables 1 and 2 show a comparison of experimental results of ETC and calculated values from (5). For solid-air, emulsion, suspension, granular and solid-solid two-phase systems the average deviation is **7.05%**, shown in table 1 and for Al-air system the average deviation is **6.49%**, shown in table 2, respectively. It is found that Jagjiwanram & Singh R model has **9.62%** error for solid-air, emulsion, suspension, granular and solid-solid two-phase systems and **13.66%** error for Al-air system, respective

Table 1

Comparison of ETC values for two-phase systems using equation (5). The thermal conductivity is in W m^{-1} K^{-1}.

S. No.	Type of the sample	Type of the sample	φ1	λ1	λ2	λe (Expt)	λe(Theo) by(5)	% Error
1	Cu/solder[a]	Cu/soldera	0.0507	398	78.1	85.2	0.916666667	1.836211
2	Cu/solder[a]	Cu/soldera	0.0996	398	78.1	92.4	0.845238095	1.592747
3	Cu/solder[a]	Cu/soldera	0.0263	398	78.1	81.7	0.955936353	2.039016
4	Cu/solder[a]	Cu/soldera	0.0286	398	78.1	82	0.952439024	2.056175
5	Cu/solder[a]	Cu/soldera	0.1029	398	78.1	92.7	0.842502697	1.805
6	Cu/solder[a]	Cu/soldera	0.2377	398	78.1	115.4	0.67677643	2.611981
7	Cu/solder[a]	Cu/soldera	0.0848	398	78.1	90.2	0.865853659	1.618712
8	Cu/solder[a]	Cu/soldera	0.1586	398	78.1	102	0.765686275	1.352822
9	Cu/solder[a]	Cu/soldera	0.2516	398	78.1	118	0.661864407	2.881643
10	cellosize/flexol[b]	cellosize/flexolb	0.3	0.616	0.161	0.235	0.685106383	8.563643
11	cellosize/polypropyllene glycol[c]	cellosize/polypropyllene glycolc	0.3	0.55	0.15	0.234	0.641025641	0.457189
12	Water/mineral Oil[c]	Water/mineral Oilc	0.4	0.611	0.149	0.292	0.510273973	0.758041
13	Selenium/poly. Glycol[d]	Selenium/poly. Glycold	0.4	5.192	0.14	0.422	0.331753555	1.895136
14	Ti.Oxide/methyl vinyl[e]	Ti.Oxide/methyl vinyle	0.25	7.985	0.174	0.461	0.377440347	22.8614
15	Graphite/water[f]	Graphite/waterf	0.11	160.5	0.666	1.132	0.588339223	10.49846
16	Graphite/water[f]	Graphite/waterf	0.17	160.5	0.666	1.439	0.462821404	18.61965
17	Selenium/polypropylene glycol[d]	Selenium/polypropylene glycold	0.3	5.208	0.14	0.316	0.443037975	1.616321
18	Selenium/polypropylene glycol[d]	Selenium/polypropylene glycold	0.4	5.208	0.14	0.423	0.330969267	1.684047
19	Water/mineral Oil[c]	Water/mineral Oilc	0.4	0.611	0.149	0.293	0.508552423	1.096751

20	cellosize/F plasticizer[b]	cellosize/F plasticizerb	0.1	0.551	0.166	0.19	0.873684211	0.338997
21	cellosize/F plasticizer[b]	cellosize/F plasticizerb	0.1	0.577	0.19	0.21	0.904761905	2.905614
22	cellosize/F plasticizer[b]	cellosize/F plasticizerb	0.3	0.467	0.135	0.18	0.75	15.73142
23	cellosize/F plasticizer[b]	cellosize/F plasticizerb	0.3	0.551	0.166	0.236	0.703389831	7.333664
24	cellosize/F plasticizer[b]	cellosize/F plasticizerb	0.3	0.577	0.19	0.256	0.7421875	10.49953
25	cellosize/polypropylene glycol[b]	cellosize/polypropylene glycolb	0.1	0.577	0.154	0.18	0.855555556	0.442766
26	cellosize/polypropylene glycol[b]	cellosize/polypropylene glycolb	0.3	0.467	0.11	0.157	0.700636943	13.99819
27	Lead powder/Si rubber[g]	Lead powder/Si rubberg	0.16	34.72	0.385	0.651	0.591397849	0.257117
28	Lead powder/Si rubber[g]	Lead powder/Si rubberg	0.24	34.72	0.385	0.862	0.446635731	8.376528
29	Bi powder/Si rubber[g]	Bi powder/Si rubberg	0.16	8.33	0.385	0.591	0.65143824	2.187911
30	Bi powder/Si rubber[g]	Bi powder/Si rubberg	0.24	8.33	0.385	0.734	0.524523161	1.224143
31	ZnO/methyl vinyl[e]	ZnO/methyl vinyle	0.15	23.1	0.1743	0.378	0.461111111	23.40588
32	TiO/methyl vinyl[e]	TiO/methyl vinyle	0.25	7.81	0.174	0.462	0.376623377	23.12635
33	ZnO/synthetic rubber[e]	ZnO/synthetic rubbere	0.21	23.1	0.168	0.43	0.390697674	24.86585
34	TiO/synthetic rubber[e]	TiO/synthetic rubbere	0.18	7.81	0.168	0.359	0.467966574	19.24147

Average deviation 7.05%

Table 2

Comparison of ETC values for two-phase systems using equation (5). The thermal conductivity is in $W\ m^{-1}\ K^{-1}$.

S. No.	Type of the sample	ϕ_1	λ_1	λ_2	λ_e(Expt)	λ_e(Theo)by(5)	% Error
1	Al/air[h]	0.029	218	0.026	2.7	2.698591029	0.05218412
2	Al/air[h]	0.095	218	0.026	6.7	8.219435061	22.6781352
3	Al/air[h]	0.091	218	0.026	6.7	7.43297741	10.9399613
4	Al/air[h]	0.094	218	0.026	6.9	8.00872482	16.0684757
5	Al/air[h]	0.028	218	0.026	2.5	2.664272083	6.57088333

Average deviation 6.49%

Conclusions

1. The model is capable of predicting ETCs close to the experimental values even for mixtures of higher conductivity ratios and high porosities, whereas one may find that other models give higher deviations in those situations.

2. The proposed model is also applicable for systems having a high ETC ratio of the solid and fluid phases.

3. This model is very simple but powerful enough without compromising on the results.

4. This model generalizes the work of Singh et al [10], who treated a three-dimensional cubic array with spherical particales.

References

1) Babanov A A, Sov Phys Tech Phys, 2 (1957) 476.
2) Brailsford A D & Major K G, Br J Appl Phys, 15 (1964) 313.
3) Pande R N, Indian J Pure & Appl Phys, 26 (1988) 691.
4) Hadley G R, Int J of Heat and Mass transfer, 20 (1986) 909.
5) Oshima N & Watari N, Japan Soc Mech Eng Int J, 32 (1989) 225.
6) Verma L S, Shrotriya A K, Singh R & Chaudhary D R, J Phys D: Appl Phys, 24 (1991) 208.
7) Cheng S C & Vachon R I Int J Heat and Mass transfer, 12 (1969) 249.
8) Jagjiwanram & Singh R Indian J Eng Mater Sci, 27 (2004) 373.
9) Tareev B Physics of Dielectric Materials, (1975) 128.
10) Singh K J, Singh Ramvir & Chaudhary D R, J. Phys D: Appl Phys, 31 (1998) 1681.