

# Velocity based Robot Motion Planning in Dynamic Environments

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## Abstract

Robot Motion Planning (RMP) has been a thrust area of research in computing due to its complexity, since RMP in dynamic environments for a point robot with bounded velocity is an NP-hard problem. This paper is a critical study of the velocity based RMP in dynamic environments. The motion planning in dynamic environments with moving obstacles has been classified into two categories: a) movements of the obstacles are completely known to the robot b) movements of the obstacles are completely unknown to the robot. In the first category, for computing the trajectories of a robot moving in a time-varying environment, the concept of Velocity Obstacles (VO) is utilized, which represents the robot's velocities that would cause a collision with an obstacle at some future time. An avoidance maneuver is computed by selecting velocities that are outside of the VO. In the second category motion planning, obstacle avoidance and safety of motion is implemented using the concept of Director Circle (DC).

**Keywords:** Velocity, Dynamic Environment, Obstacles, Motion planning.

## Introduction

Many efforts have been conducted in robotic research for solving the fundamental problem of motion planning which consists of generating a collision-free path between start and goal position for a robot in a static and completely known environment, where there could be obstacles. Mobile robot motion planning in dynamic environment has been studied extensively and there is a strong evidence that a complete planner (i.e. one that finds a path whenever one exists and report that no one exists otherwise) will take time exponential in the number of degrees of freedom of the robot and the algorithm belongs to a class of NP-Complete.<sup>1</sup> In recent years, a class of motion planning problems that has got more attention is the motion planning in dynamic environment with moving obstacles and moving targets. It was shown that dynamic motion planning for a point in the plane, with bounded velocity and arbitrary many obstacles, is intractable and NP-Hard.<sup>2</sup>

## Aim of the Study

An optimal and safe motion planning of the Robot in the known and unknown Dynamic Environments on the basis of Velocity.

## Literature Review

Motion planning in dynamic environments was originally addressed by adding the time dimension to the robot's configuration space, assuming bounded velocity and known trajectories of the obstacles.<sup>[2]</sup> solved the planar problem for a polygonal robot among many moving polygonal obstacles by searching a visibility graph in the configuration space<sup>3</sup> discretized the configuration-time space to result in a sequence of configuration space slices at successive time intervals. This method solves the static planning problem at every slice and joins adjacent solutions.<sup>4</sup> used a cell decomposition to represent the configuration-time space, and joined empty cells to connect start to goal.

Another approach to dynamic motion planning is to decompose the problem into smaller problems: path planning and velocity planning. This method first computes a feasible path among static obstacles, and represent it as a parametric curve in the arc length. Then, the intersections of the moving obstacles with the path are represented as a forbidden regions in an arc length-time plane. The velocity along the path is chosen to avoid the forbidden region<sup>5</sup> selected both path and velocity profile using a visibility graph approach<sup>6</sup> developed independently a similar approach for two cooperating robots, and compared the effect of delay and velocity reduction on motion time<sup>7</sup> considered acceleration bounds, and used a search in a state-time space to compute the velocity profile

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yielding a minimum-time trajectory.<sup>8</sup> considered adjacent paths that could be reached from the nominal path when the nominal path becomes blocked by the moving obstacle.

A different approach consists of generating the accessibility graph of the environment, which is an extension of visibility graph.<sup>9</sup> defined it as the locus of points on the obstacles which are reachable by the robot moving at maximum speed. These points form the collision front, that can be linked together to construct a path from start to goal. The accessibility graph has the property that, if the robot moves faster than the obstacles, the path computed by searching the graph is time minimal. This concept was extended to the case of slowly moving robots and transient obstacles, i.e, obstacles that could appear and disappear in the environment.

Robot motion planning in dynamic and uncertain environments (DUE) require interpretation about future developments and uncertainties about the states of the dynamic agents and obstacles. Dynamic programming (DP) is a solution approach for solving chronological decision models based on inductive computation. DP has been employed here to account for future information assimilation and the quality of that information in the planning process. DP provides a valid basis for compiling planning results into strategies for control, as well as for learning such strategies when the system being controlled is incompletely known. Stochastic dynamic programming (SDP) generates an approximate solution to RMP in DUE.<sup>10</sup> Unlike in the case for SDP, the bounds of the time horizon in partially closed-loop receding horizon control (PCLRHC) are restricted.<sup>11</sup>

Probabilistic robotics is a paradigm for robot programming in motion planning.<sup>12</sup> The probabilistic model acknowledges the inherent uncertainty in robot observation relying on representations of uncertainty when determining what is to be done in the next instant of time. The central conjecture is that probabilistic methods of robotics do better in complex real-world applications than methods that overlook a robot's uncertainty.

The motion planning problem in dynamic environment with moving obstacles can be classified into two categories as follows:

- Movements of obstacles are completely known to the robot
- Movements of obstacles are completely unknown to the robot.

To make the problem computationally tractable, the following assumptions are made :a)The environment is restricted to the plane, b) The robots and the obstacles are modeled by circles, c)The information about the environment is complete.

#### Definition (using Mayer's notation). 6.

Mathematically, the planning problem consists of computing the control  $u(t) \in u$  in  $t_0 \leq t \leq t_f$  that minimize the performance index

$$J = \varphi(X(t_f), t_f) + \int_{t_0}^{t_f} L(x, u, t) dt \quad (1)$$

subject to the two set of constraints

#### Kinematic Constraints

$$\text{initial manifold: } \Gamma(x(t_0), t_0) = 0 \quad (2)$$

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$$\text{final manifold: } \Omega(x(t_f), t_f) = 0 \quad (3)$$

obstacles:

$$\psi: \cup_{i=1}^n [S_i(x_t, t) = 0] \quad (4)$$

#### Dynamic Constraints

robot dynamics:

$$\dot{x} = F(x, u) = f(x) + g(x)u \quad (5)$$

#### Actuator Constraints

$$u_i(\min) \leq u_i \leq u_i(\max)$$

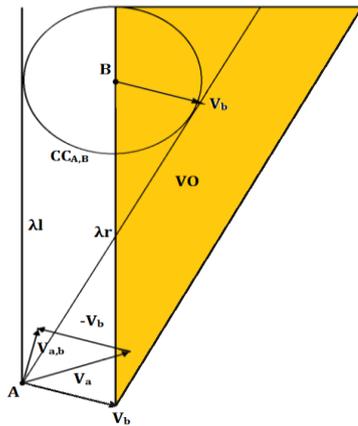
The complete problem of motion planning can be decomposed in the two separate problem : the Kinematic problem and the Dynamic problem. The kinematic problem consist of finding a trajectory that takes into account position and velocity of the obstacles as well as an approximation of the dynamic constraints of the robot. The dynamic problem consists of computing an optimal trajectory that satisfies the full set of kinematic and dynamic constraints, and is in a neighborhood of the solution to the kinematic problem

Therefore, the approach to the solution of the complete motion planning problem consists of two steps. The first step compute a kinematic trajectory that solves the kinematic problem. The second step uses a dynamic optimization to optimize motion time subject to the dynamic constraints, using the kinematic trajectory as an initial guess.

For computing the trajectories of a robots moving in a time-varying environment by using the concept of Velocity

#### Obstacles (VO),

Which represents the robot's velocities that would cause a collision with an obstacles at some future time. An avoidance maneuver is computed by selecting velocities that are outside of the velocity obstacles. To ensure that the maneuver is dynamically feasible, robot dynamics and actuator constraints are mapped into the robot velocity space. A trajectory consists of a sequence of such avoidance maneuvers, computed by searching over a tree of avoidance maneuvers generated at discrete time intervals. For on-line applications, the tree is pruned using a heuristic search designed to achieve a prioritized set of objectives, such as avoiding collisions, searching the goal, maximizing speed or computing trajectories with desirable topology. To evaluate the quality of these trajectories, they are compared to the trajectories computed using a dynamic optimization, which are not bounded by the velocity obstacles approach. The dynamic optimization is based on Pontryagin's minimum principle and uses a gradient descent method. It works for an obstacle that moves with a constant linear velocity



Velocity Obstacles

**Figure.1** The advantage of this approach are as follows

it permits an efficient geometric representation of potential avoidance maneuvers of the moving obstacles, b) any number of moving obstacles can be avoided by considering the union of their VO's, c) it unifies the avoidance of moving as well as stationary obstacles, and d) it allows the simple consideration of robot dynamics and actuator constraints.

**Movements of obstacles are completely unknown to the robots**

**Problem assumptions. 4.**

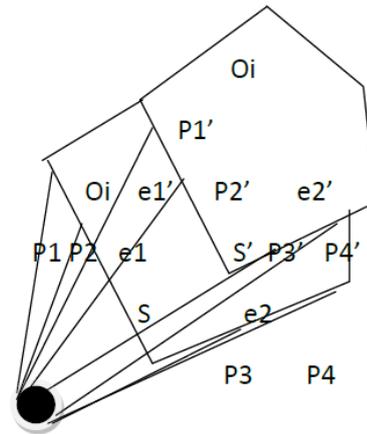
**Robot:** The robot (R) is circular and omni directional. It can move with maximum speed of  $\bar{V}_R$ , which should be known at the beginning of planning. It is equipped with range sensors encircled around it.

**Obstacles:** Each obstacle is represented with OBi. Obstacles may be static or dynamic, with the speed of  $\bar{V}_{oi}$ . Their speed be set at the beginning, and proportional to the robot's velocity. Obstacles can be concave or convex polygonal shape and their speed vector is unknown to the robot.

**Target:** It is assumed that only one target exists in the problem, within the defined border. Velocity of the target is shown with  $\bar{V}_T$ , and it is proportional to the robot's velocity.

Since the velocity and position of obstacles is unknown for the robots, it must be equipped with detectors or range sensors to acquire necessary information. This is done by performing a visibility scan and detecting visible obstacles vertices. Because of the dynamic nature of the problem in which obstacles may mask each other or follow unknown directions, they are not just identifiable by their vertices. Therefore, a theoretical land mark is necessary to track the obstacles. Since two visible edges of an obstacles will definitely intersect, their intersection can be computed as a landmark point. This point will help the robot in determining the obstacles speed vector

As show in Figure 2, suppose that e1 and e2 are the edge of obstacle  $O_i$ . The robot senses P1 and P2 on edge e1, and P3 and P4 on edge e2 at time t1.



**Figure. 2**

**Finding velocity of obstacles**

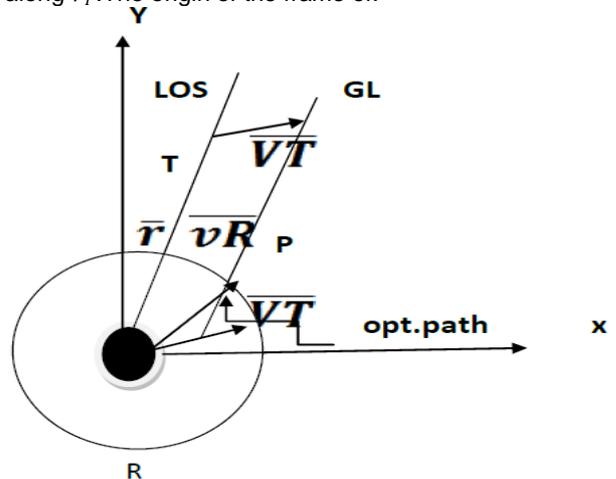
At the next time interval, t2 the points P1' and P2' are sensed on edge e1', and points P3' and P4' on edge e2'. The slope of each edge can be calculated based on this points. The algorithm looks for two equal slope among visible edges at time interval [t1 t2]. In this case S1 and S2 are the landmark points. The velocity vectors  $\bar{V}_{x0}$  and  $\bar{V}_{y0}$  of the obstacle would be

$$\bar{V}_{x0} = x_s' - x_s$$

$$\bar{V}_{y0} = y_s' - y_s$$

**Target following**

A method for the interception of the moving targets in the presence of obstacles is proposed. Define a line-of-sight(LOS) as the relative position vector,  $\bar{r}$ , connecting the robots to the target, as shown in Figure 3. The parallel navigation law state that the direction of LOS should remain constant relative to the non- rotating frame, while robot approaches the target. The relative velocity,  $r'$  between the robot and the target should remain parallel to the LOS(i.e.,  $\bar{r}$ ) at all time. The parallel navigation law is expressed by the following two relationship  $\bar{r} \times r' = 0$ ,  $\bar{r} \cdot r' < 0$ , the first equation guarantees that the LOS and relative velocity remain parallel and other ensures that interceptor is not receding from the target. Suppose the target T moves along  $\bar{V}_T$ . The origin of the frame of.



**Figure. 3** Finding optimum path towards target

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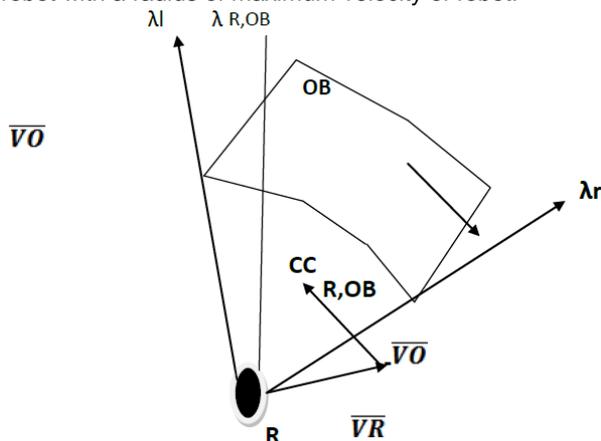
coordinates is located on the robot's center to show the instantaneous relative position of the target Hence vectors  $\vec{r}$  and  $\vec{V}_T$ . are known to the robots. The end points of the velocity vectors present the position of the target and robot, after one time interval has elapsed. GL is the guidance line is a semi-line parallel to LOS. If the end point of  $\vec{V}_R$  falls on this semi-line, the direction of LOS would remain constant, and position matching between the robots and the moving target is guaranteed. The goal of the planner is to obtain a robot- velocity command for the next command instant, according to the parallel navigation law. First draw a circle with radius  $\|\vec{V}_R\|$  is drawn around the robot. This circle intersects the semi-line GL at P. This is the end point of  $\vec{V}_R$ . It is evident that  $\vec{V}_R$  must be at least equal to target velocity in order to reach target.

**Obstacle avoidance by using Directive Circle**

The Directive Circle (DC) decides the acceptable and forbidden direction of the robot to move. At any instant, the robot recognizes the velocities of obstacles using the method explained in the previous part. Figure 3 illustrates the creation of DC. Consider the robot, R and obstacle OB which is concave. The velocity of the obstacle is  $\vec{V}_O$ . We define the Collision Cone,  $CC_{R,OB}$ , as the set of relative velocities between the colliding R and OB

$CC_{R,OB} = \{ V_{R,OB}, \lambda_{R,OB} \cap OB \neq \emptyset \}$   
 where  $V_{R,OB} = \vec{V}_R - \vec{V}_O$  is the relative velocity of R with respect to OB, and  $\lambda_{R,OB}$  is the direction of  $\vec{V}_{R,OB}$ .  $\lambda_r$  and  $\lambda_t$  represent the two tangent to the obstacle. For every robot/obstacle pair, there is a unique Collision Cone.

In DC, we find the forbidden directions as set of directions in  $\lambda_{R,OB}$  lied on  $CC_{R,OB}$ . For forming the DC, the position of the robot along  $-\vec{V}_O$  must be shifted. Then we draw a circle C around the robot with a radius of maximum velocity of robot.



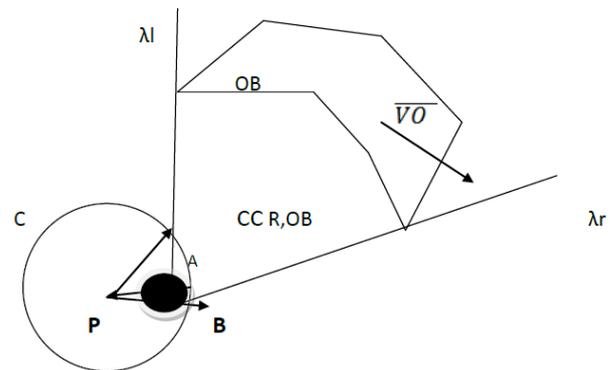
**Figure 4. Collision cone CCR, OB**

Geometrically, one case from four cases may occur

1. the circle intersect with  $\lambda_r$
2. the circle intersect with  $\lambda_t$
3. the circle intersect both  $\lambda_r$  and  $\lambda_t$
4. the circle intersect none of them

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As shown in figure 4, the circle C has intersection with  $\lambda_t$  at the point A and  $\lambda_r$  with B. If the slope of  $\lambda_{R,OB}$  falls between the slope of PA and PB vectors, the robot will collide with the obstacles. C is the Directive Circle. If the circle C doesn't have any intersection points with the tangent  $\lambda_r$  and  $\lambda_t$ , DC doesn't have any forbidden zone, and robot can freely move along the calculated optimal direction to the target. If whole circle fall in  $CC_{R,OB}$ , then all possible directions are forbidden and the robot should stop. This condition usually happen when the robot is in a cluttered space, and obstacles have surrounded the robot tightly.



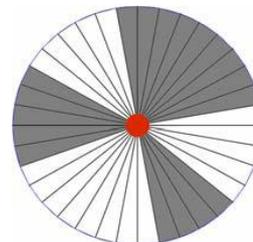
**Figure 5. Forbidden zone and Directive Circle (DC)**

The DC must be built for all moving or static obstacles. Let  $F_i$  show the forbidden area for each obstacles  $i$ . Then total DC for all obstacles is computed by superimposition all partial DCs as  $F = \bigcup_{i=1}^m F_i$

**Optimal solution search.**

By following the algorithm, a set of collision free path are found for the center of robot. Our purpose is to find the best collision-free path. Consider the DC of the problem shown in Figure 6. By dividing the DC into N parts, there are N direction ( $r_i$ ) to select. Some of these path are located in forbidden area and cannot be selected.

**Acceptable directions are selected based on these condition:**



**Figure 6: Total DC for 3 disjoint obstacles**

1. The optimum path falls in acceptable area
2. The optimum path falls in forbidden area

If the first condition occurs, the algorithm chooses the optimum path calculated by the previous stated method. If the second one happens, slope of angles of optimum path is determined and shown with  $\Theta'$ . Then, the slope all acceptable direction are measured and named as  $\Theta_i$ , the gap  $g = |\Theta_i - \Theta'|$ ; for all  $\Theta_i \in \Theta$

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In order to find the optimum direction, the probability of each direction is computed by

$$P(n) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \quad (6)$$

in which  $x_i = g_i \bmod \square$ ,  $0 \leq x_i \leq \square$ , so the direction with maximum probability is selected as follows

$$\bar{v} = \{ r_i \mid P(r_i) \geq P(r_j), \text{ for all } i \neq j \} \quad (7)$$

This DC method also guarantee and check the safety of the robot path in order to avoid collisions.

### Conclusion

The VO is the vector sum of collision cone with the velocity vector of the obstacle when the robot knows about the movements of obstacles completely. The VO represents a region in the velocity space of the robot that would lead to a collision with the obstacle within a time horizon. Obstacle avoidance is carried out by creating a set of reachable avoidance velocities defined by the dynamic constraints of the vehicle. The approach consists of refining the trajectory with a dynamic optimization that, subject to the robot's dynamics, actuator constraints and time-varying obstacle constraints, minimizes motion time.

At each and every iteration, the set of all collision free directions are computed using velocity vectors of the robot related to each obstacle, forming the Directive Circle (DC), which is the main concept of the method when the movements of the obstacles are unknown to the robot. Then, the best direction near to the optimal direction to the target is selected from the DC, which prevents the robot from being trapped in local minima. The movements of the robots are governed by the exponential stabilizing control method that provides a proper motion at each step, while considering the robot's kinematic constraints.

The robot is able to achieve the target safely at a desired orientation.

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