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A New Application of Mahgoub Transform for Solving Linear Volterra Integral Equations

Sudhanshu Aggarwal
Assistant Professor,
Deptt.of Mathematics,
National P.G. College,
Barhalganj, Gorakhpur

Raman Chauhan
Assistant Professor,
Deptt.of Mathematics,
Noida Institute of Engineering & Technology,
Greater Noida, U.P.

Nidhi Sharma
Assistant Professor,
Deptt.of Mathematics,
Noida Institute of Engineering & Technology,
Greater Noida, U.P.

Abstract

In this paper, we used Mahgoub transform for solving linear Volterra integral equations and some applications are given in order to demonstrate the effectiveness of Mahgoub transform for solving linear Volterra integral equations.

Keywords: Linear Volterra Integral Equation, Mahgoub Transform, Convolution Theorem, Inverse Mahgoub Transform.

Introduction

Volterra examined the linear Volterra integral equation of the form.[1-5]

$u(x) = f(x) + \lambda \int_0^x k(x,t)u(t)dt$ .... (1)

where the unknown function $u(x)$, that will be determined, occurs inside and outside the integral sign. The kernel $k(x,t)$and the function $f(x)$ are given real-valued functions, and λ is a parameter. The Volterra integral equations appear in many physical applications such as neutron diffusion and biological species coexisting together with increasing and decreasing rates of generating.

The Mahgoub transform of the function $F(t)$is defined as [6]:

$$M\{F(t)\} = v\int_0^\infty F(t)e^{-vt}dt = H(v), t \geq 0, k_1 \leq v \leq k_2$$

where M is Mahgoub transform operator.

The Mahgoub transform of the function $F(t)$ for $t \geq 0$ exist if $F(t)$ is piecewise continuous and of exponential order. These conditions are only sufficient conditions for the existence of Mahgoub transform of the function $F(t)$. Mahgoub and Alshikh [7] used Mahgoub transform for solving partial differential equations.Fadhil [8] discussed the convolution for Kamal and Mahgoub transforms.Taha et.al.[9] gave the dualities between Kamal and Mahgoub integral transforms and some famous integral transforms.

Aim of the Study

The aim of this work is to establish exact solutions for linear Volterra integral equation using Mahgoub transform without large computational work.

Mahgoub transform of some elementary functions [6, 8]

S.N.	$F(t)$	$M\{F(t)\} = H(v)$
1.	1	1
2.	t	$\frac{1}{v}$
3.	t^2	$\frac{2!}{v^2}$
4.	$t^n, n \geq 0$	$\frac{n!}{v^n}$
5.	e^{at}	$\frac{v}{v-a}$
6.	$sinat$	$\frac{av}{v^2+a^2}$
7.	$cosat$	$\frac{v^2}{v^2+a^2}$
8.	$sinhat$	$\frac{av}{v^2-a^2}$
9.	$coshat$	$\frac{v^2}{v^2-a^2}$

Mahgoub transform of the derivatives of the function $F(t)$ [6, 8, 9]

If $M\{F(t)\} = H(v)$ then

a) $M\{F'(t)\} = vH(v) - vF(0)$

b) $M\{F''(t)\} = v^2H(v) - vF'(0) - v^2F(0)$

c) $M\{F^{(n)}(t)\} = v^nH(v) - v^nF(0) - v^{n-1}F'(0) \dots\dots - vF^{(n-1)}(0)$

Convolution of two functions [8]

Convolution of two functions $F(t)$ and $G(t)$ is denoted by $F(t) * G(t)$ and it is defined by

$$F(t) * G(t) = F * G = \int_0^t F(x)G(t-x)dx$$
$$= \int_0^t F(t-x)G(x)dx$$

Convolution theorem for Mahgoub transforms [8]

If $M\{F(t)\} = H(v)$ and $M\{G(t)\} = I(v)$ then

$$M\{F(t) * G(t)\} = \frac{1}{v}M\{F(t)\}M\{G(t)\} = \frac{1}{v}H(v)I(v)$$

Inverse Mahgoub Transform

If $M\{F(t)\} = H(v)$ then $F(t)$ is called the inverse Mahgoub transform of $H(v)$ and mathematically it is defined as $F(t) = M^{-1}\{H(v)\}$

where M^{-1} is the inverse Mahgoub transform operator.

Inverse Mahgoub transform of Some Elementary Functions

S.N.	$H(v)$	$F(t) = M^{-1}\{H(v)\}$
1.	1	1
2.	$\frac{1}{v}$	t
3.	$\frac{1}{v^2}$	$\frac{t^2}{2!}$
4.	$\frac{1}{v^n}, n \geq 0$	$\frac{t^n}{n!}$
5.	$\frac{v}{v-a}$	e^{at}
6.	$\frac{v}{v^2+a^2}$	$\frac{sinat}{a}$
7.	$\frac{v^2}{v^2+a^2}$	$cosat$
8.	$\frac{v}{v^2-a^2}$	$\frac{sinhat}{a}$
9.	$\frac{v^2}{v^2-a^2}$	$coshat$

Mahgoub Transforms for linear Volterra Integral Equations

In this work we will assume that the kernel $k(x,t)$ of (1) is a difference kernel that can be expressed by the difference $(x-t)$. The linear Volterra integral equatin (1) can thus be expressed as

$u(x) = f(x) + \lambda \int_0^x k(x-t)u(t)dt$ …………… (2)

Applying the Mahgoub transform to both sides of(2), we have

$M\{u(x)\} = M\{f(x)\} + \lambda M\{\int_0^x k(x-t)u(t)dt\}$..(3)

Using convolution theorem of Mahgoub transform, we have

$M\{u(x)\} = M\{f(x)\} + \frac{\lambda}{v}M\{k(x)\}M\{u(x)\}$……….. (4)

Operating inverse Mahgoub transform on both sides of(4), we have

$u(x) = f(x) + \lambda M^{-1}\left\{\frac{1}{v}M\{k(x)\}M\{u(x)\}\right\}$……….. (5)

which is the required solution of (2).

Applications

In this section, some applications are given in order to demonstrate the effectiveness of Mahgoub transform for solving linear Volterra integral equations.

Application: 1

Consider linear Volterra integral equation with $\lambda = -1$

$u(x) = 1 - \int_0^x (x-t)\,u(t)dt$………… (6)

Applying the Mahgoub transform to both sides of(6), we have

$M\{u(x)\} = 1 - M\{\int_0^x (x-t)\,u(t)dt\}$…………. (7)

Using convolution theorem of Mahgoub transform on (7), we have

$M\{u(x)\} = \frac{v^2}{1+v^2}$…………. (8)

Operating inverse Mahgoub transform on both sides of(8), we have

$u(x) = M^{-1}\left\{\frac{v^2}{1+v^2}\right\} = cosx$…………….. (9)

which is the required exact solution of (6).

Application : 2

Consider linear Volterra integral equation with $\lambda = -1$

$u(x) = cosx + sinx - \int_0^x u(t)\,dt$…… (10)

Applying the Mahgoub transform to both sides of(10), we have

$M\{u(x)\} = \frac{v^2}{1+v^2} + \frac{v}{1+v^2} - M\{\int_0^x u(t)\,dt\}$…………. (11)

Using convolution theorem of Mahgoub transform on(11), we have

$M\{u(x)\} = \frac{v^2}{1+v^2}$…………. (12)

Operating inverse Mahgoub transform on both sides of(12), we have

$u(x) = M^{-1}\left\{\frac{v^2}{1+v^2}\right\} = cosx$…………….. (13)

which is the required exact solution of (10).

Application: 3

Consider linear Volterra integral equation with $\lambda = 1$

$u(x) = 1 - x + \int_0^x (x-t)u(t)\,dt$…… (14)

Applying the Mahgoub transform to both sides of(14), we have

$M\{u(x)\} = 1 - \frac{1}{v} + M\{\int_0^x (x-t)u(t)\,dt\}$…. (15)

Using convolution theorem of Mahgoub Transform on(15), we have

$M\{u(x)\} = \frac{v}{v+1}$…………. (16)

Operating inverse Mahgoub transform on both sides of(16), we have

$u(x) = M^{-1}\left\{\frac{v}{v+1}\right\} = e^{-x}$ …………… (17)

which is the required exact solution of (14).

Application: 4

Consider linear Volterra integral equation with $\lambda = -1$

$u(x) = x - \int_0^x (x-t)u(t)\,dt$…… (18)

Applying the Mahgoub transform to both sides of(18), we have

$M\{u(x)\} = \frac{1}{v} - M\{\int_0^x (x-t)u(t)\,dt\}$…. (19)

Using convolution theorem of Mahgoub transform on(19), we have

$M\{u(x)\} = \frac{v}{1+v^2}$............ (20)

Operating inverse Mahgoub transform on both sides of(20), we have

$u(x) = M^{-1}\left\{\frac{v}{1+v^2}\right\} = sinx$............ (21)

which is the required exact solution of (18).

Application: 5

Consider linear Volterra integral equation with $\lambda = 1$

$$u(x) = 1 - \frac{x^2}{2} + \int_0^x u(t)\,dt......(22)$$

Applying the Mahgoub transform to both sides of (22), we have

$M\{u(x)\} = 1 - \frac{1}{v^2} + M\left\{\int_0^x u(t)\,dt\right\}$.... (23)

Using convolution theorem of Mahgoub transform on(23), we have

$M\{u(x)\} = 1 + \frac{1}{v}$ (24)

Operating inverse Mahgoub transform on both sides of(16), we have

$$u(x) = M^{-1}\{1\} + M^{-1}\left\{\frac{1}{v}\right\}$$
$$= 1 + x............... (25)$$

which is the required exact solution of (22).

Conclusion

In this paper, we have successfully developed the Mahgoub transform for solving linear Volterra integral equations. The given applications showed that the exact solution have been obtained using very less computational work and spending a very little time. The proposed scheme can be applied for other linear Volterra integral equations and their system.

References

1. *Rahman, M. (2007) Integral equations and their application, with press, Southampton, Boston.*
2. *Linz, P. (1984) Analytical and numerical methods for Volterra equations, SIAM, Philadelphia.*
3. *Jerri, A. J. (1999) Introduction to integral equations with applications, Wiley, New York.*
4. *Kanwal, R. (1997) linear integral equations, Birkhauser, Berlin.*
5. *Wazwaz, A.M. (1997) A first course in integral equations, World scientific, Singapore.*
6. *Mahgoub, M.A.M. (2016) The new integral transform "Mahgoub Transform", Advances in theoretical and applied mathematics, 11(4), 391-398.*
7. *Mahgoub,M.A.M. and Alshikh,A.A. (2017) An application of new transform "Mahgoub Transform" to partial differential equations, Mathematical theory and Modeling, 7(1), 7-9.*
8. *Fadhil, R.A. (2017) Convolution for Kamal and Mahgoub transforms, Bulletin of mathematics and statistics research, 5(4), 11-16.*
9. *Taha, N.E.H., Nuruddeen, R.I., Abdelilah, K. and Hassan, S. (2017) Dualities between "Kamal & Mahgoub Integral Transforms" and "Some Famous Integral Transforms", British Journal of Applied Science & Technology, 20(3), 1-8.*