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A Plane Symmetric Universe Filled With Viscous Fluid in a Self-Creation Theory of Gravitation

Abstract

In recent years a considerable interest has been focused by cosmologists in formation of alternative theories of gravitations. In the present paper, a plane symmetric cosmological model filled with viscous fluid has been derived in the Barber's second self creation theory of gravitation. We have also discussed and studied some physical and geometrical properties of this model.

Keywords: Plane symmetric model, Barber's second self creation theory and viscous fluid.

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Introduction

In 1916, Einstein's general theory of relativity has provided a sophisticated theory of gravitation. It has been very successful in describing gravitational phenomena. This theory has also served as a basis for models of the universe. Since Einstein himself pointed out that general relativity does not account satisfactorily for the inertial properties of matter, i.e. Mach's principle is not substantiated by general relativity. In recent years, there have been some interesting attempts to generalize the general theory of relativity by incorporating certain desired features which are lacking in the original theory.

Brans-Dicke [1] introduced a scalar-tensor theory of gravitation involving a scalar function in addition to the familiar general relativistic metric tensor. In this theory the scalar field has the dimension of inverse of the gravitational constant and its role is confined to its effects on gravitational field equations. This theory does not allow the scalar field to interact with fundamental principles and photons. Barber [2] proposed two self-creation cosmologies by modifying the Brans and Dicke [1] theory and general relativity.

These modified theories create the universe out of self contained gravitational, scalar and matter fields. Brans [3] has pointed out that Barber's first theory is not only in disagreement with experiment, but is actually inconsistent. Hence, Barber's first theory is not accepted because this theory violets the equivalence principle. Barber's second self-creation theory is a modification of Einstein's theory of general relativity to a variable G-theory and predicts local effects. In this theory the scalar field does not directly gravitate, but simply divides the matter tensor, acting as a reciprocal gravitational constant. It is postulated that the scalar field couples to the trace of the energy momentum tensor.

Hence the field equations in Barber's second self creation theory are:

$$G_{ij} = -\frac{8\pi}{\phi} T_{ij} \tag{1.1}$$

and the scalar field ϕ satisfies the equation

$$\Box \phi = \frac{8\pi}{3} \lambda T \tag{1.2}$$

where $G_{ij} = R_{ij} - \frac{1}{2} R g_{ij}$ is an Einstein tensor, T_{ij} is the stress energy tensor of the matter, T is the trace of the stress energy tensor describing all non-gravitational and non-scalar field energy, and $\Box\phi = \phi^{k}_{;k}$ is the invariant d' Alembertian. Barber's scalar ϕ is a function of t due to the nature of space-time which plays the role analogous to the reciprocal of Newtonian gravitational cpnstant i.e. $\phi = G^{-1}$. λ is a coupling constant to be determined from experiments and the measurements of the deflection of light restricts the value of the coupling constant to $|\lambda| < 10^{-1}$.A comparison with Einstein's equations shows that the Barber's theory goes over to general relativity in every respect in case of coupling constant $\lambda \to 0$, ϕ = constant = G^{-1}. With a small value of λ there would have been a violent period of matter creation in the earliest stages of the Big-Bang. However, such a view is probably unable to explain all the observed helium abundance and other mechanisms. For detailed discussion of the self creation theories of gravitation one may refer to the work of Barber [2]. Barber [2] and Soleng [4] have discussed the Friedmann-Robertson-Walker (FWR) models while Reddy and Venkateswarlu [5] have studied the Bianchi type-VI$_0$ cosmological model in Barber's second theory of gravitation, both in vacuum as well as in the presence of perfect fluid with pressure equal to energy density.

Shanthi and Rao [6] have studied Bianchi type-II, III cosmological models in self creation theory, both in vacuum as well as in the presence of stiff fluid. Ram and Singh [7, 8] discussed spatially homogeneous and isotropic R-W and Bianchi type-II models of the universe in Barber's second self-creation theory of gravitation in the presence of perfect fluid by using the 'gamma-law' equation of state.

Recently, Rai et al. [9] have presented an exact solution of spatially homogeneous and anisotropic cylindrically symmetric cosmological model in Barber's second self-creation theory of gravitation in the presence of perfect fluid with pressure equal to energy density. Rai et al. [10] also have presented an exact solution of spatially homogeneous and anisotropic plane symmetric cosmological model in Barber's second self-creation theory of gravitation, both in the presence of perfect fluid and in vacuum case.

Further, Rai *et al.* [11] have studied a plane symmetric cosmological model in self-creation cosmology in the presence of a perfect fluid distribution. In the present paper, we have studied a plane symmetric cosmological model filled with viscous fluid in Barber's second self creation theory of gravitation. We have also discussed about some physical and geometrical properties of this model.

The Field Equations

The geometry of the universe is described by the line-element

$$ds^2 = -dT^2 + Tdx^2 + T^{\frac{1+a}{2}} dy^2 + T^{\frac{1-a}{2}} dz^2 \qquad (2.1)$$

where a is a constant.

The energy momentum tensor for the viscous fluid distribution is given by [12]

$$T_i^j = (\varepsilon + p)v_i v^j + pg_i^j - \eta(v_{i;}^j + v_{;i}^j + v^j v^l v_{i;l} + v_i v^l v_{;l}^j) - (\xi - \frac{2}{3}\eta)v_{;l}^l (g_i^j + v_i v^j) \qquad (2.2)$$

together with

$$v^i v_i = -1 \qquad (2.3)$$

where p and ε are the pressure and density respectively. η and ξ are the two coefficients of viscosity, v^i are the components of the fluid four velocity and semicolons signifies covariant differentiation. The coordinates are assumed to be co-moving so that

$$v^1 = v^2 = v^3 = 0 \text{ and } v^4 = 1$$

The pressure *p* and density ε in the model (2.1) are given by

$$8\pi p = \frac{(5-a^2)}{16T^2}\left[K_1 T^{\sqrt{\frac{\lambda(a^2-5)}{24}}} + K_2 T^{-\sqrt{\frac{\lambda(a^2-5)}{24}}}\right] + \frac{8\pi\xi}{T} \qquad (2.4)$$

and

$$8\pi \varepsilon = \frac{(5-a^2)}{16T^2}\left[K_1 T^{\sqrt{\frac{\lambda(a^2-5)}{24}}} + K_2 T^{-\sqrt{\frac{\lambda(a^2-5)}{24}}}\right] \qquad (2.5)$$

Also scalar field ϕ is given by

$$\phi = K_1 T^{\sqrt{\frac{\lambda(a^2-5)}{24}}} + K_2 T^{-\sqrt{\frac{\lambda(a^2-5)}{24}}} \qquad (2.6)$$

where K_1 and K_2 are constants.

Physical & Geometrical Properties

The volume element of the model is given by

$$V = (-g)^{\frac{1}{2}} = (-T^2)^{\frac{1}{2}} = T \qquad (3.1)$$

Here volume is directly proportional to time. If the time increases then volume increases (i.e. the models are expanding with time) and if the time decreases then volume decreases (i.e. the models are contracting). The reality conditions [13]

1. $\varepsilon + p > 0$

and

2. $\varepsilon + 3p > 0$

Impose restrictions on the time during which the model exists.

The flow vector v^i of the distribution for the model (2.1) is given by

$$v^1 = v^2 = v^3 = 0 \text{ and } v^4 = 1 .$$

Obviously $$v_{;j}^i v^j = 0 \qquad (3.2)$$

Hence the flow is geodesic.
The rotation tensor $\omega_{ij} = v_{i;j} - v_{j;i}$ is zero. Thus the fluid filling the universe is non-rotational.

The expansion scalar $\theta = \frac{1}{3} v^i_{;i}$ is given by

$$\theta = \frac{1}{3T} \tag{3.3}$$

Shear tensor $\sigma_{ij} = \frac{1}{2}(v_{i;j} + v_{j;i}) - \theta(g_{ij} - v_i v_j)$ is given by

$$\sigma_{11} = \frac{1}{6}$$

$$\sigma_{22} = \frac{3a-1}{12} T^{\frac{a-1}{2}}$$

$$\sigma_{33} = -\frac{(3a+1)}{12} T^{\frac{-a-1}{2}}$$

$$\sigma_{44} = 0 \tag{3.4}$$

Also the shear σ is

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{48T^2}\left[1 + 3a^2\right] \tag{3.5}$$

Deceleration parameter $q = -3\theta^2\left[\frac{d\theta}{dT} + \frac{1}{3}\theta^2\right]$ is given by

$$q = -\frac{(9T^2 + 1)}{81T^4} \tag{3.6}$$

The deceleration parameter acts as an indicator of the existence of inflation. If q > 0, the model decelerates in its standard way while q < 0, the model inflates. The present model represents a decelerate model if $T^2 < -\frac{1}{9}$ and inflationary model if $T^2 > -\frac{1}{9}$.

The surviving components of the conformal curvature tensor C^{jk}_{hi} for the line element (2.1) are

$$C^{14}_{14} = C^{23}_{23} = \frac{1}{24T^2}\left[1 + a^2\right]$$

$$C^{12}_{12} = C^{34}_{34} = -\frac{1}{48T^2}\left[1 + 6a + a^2\right] \tag{3.7}$$

$$C^{13}_{13} = C^{24}_{24} = -\frac{1}{48T^2}\left[1 - 6a + a^2\right]$$

Thus, it follows that the space-time given by equation (2.1) is of Petrov type-I. It may be observed that the model has no initial singularity.

Discussions and Conclusion

We have successfully studied a plane symmetric cosmological model filled with viscous fluid in Barber's second self creation theory of gravitation. We have also discussed about some physical and geometrical properties of this model. When $\eta = \xi = 0$, the solution reduces to a plane symmetric cosmological model filled with perfect fluid in Barber's second self-creation theory of gravitation [11]. When $\eta = \xi = \lambda = 0$, the solution reduces to well known problem of perfect fluid distribution in general relativity of gravitation with $p = \varepsilon$ [14]. The model obtained in this paper is new and like other models with empty space they may be used in the relativistic cosmology for the description of very early stages of the universe expansion.

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