

Periodic Research

Quasi $D\alpha$ - Normal Spaces, $\pi GD\alpha$ -Closed Sets and Some Functions

Abstract

In aim this paper, we introduce a new concept of quasi-normal spaces called quasi $D\alpha$ -normal spaces and obtain characterizations and preservation theorems of quasi $D\alpha$ -normal. The notion can be applied for investigating many other properties.

Keywords: $D\alpha$ -closed, $D\alpha g$ -closed $\pi g D\alpha$ -closed, $D\alpha$ -open $D\alpha g$ -open, $\pi g D\alpha$ -open sets, $\pi g D\alpha$ -closed, almost $\pi g D\alpha$ -closed, $\pi g D\alpha$ -continuous and almost $\pi g D\alpha$ -continuous functions, $D\alpha$ -normal spaces, mildly $D\alpha$ -normal spaces and quasi $D\alpha$ -normal spaces.

2010 AMS Subject classification

54D15, 54A05, 54C08.

Introduction

In this paper, we introduce the notion of $D\alpha g$ -closed, $D\alpha g$ -open, $\pi g D\alpha$ -closed, $\pi g D\alpha$ -open sets, $\pi g D\alpha$ -closed, almost $\pi g D\alpha$ -closed, $\pi g D\alpha$ -continuous and almost $\pi g D\alpha$ -continuous functions and its properties are studied. Further we introduce a new concept of quasi-normal spaces called quasi $D\alpha$ -normal spaces and obtain characterizations and preservation theorems of quasi $D\alpha$ -normal.

Aim of the Study

In aim this paper, we introduce a new class of sets called $D\alpha g$ -closed, $\pi g D\alpha$ -closed sets and its properties are studied and we introduce a new concept of quasi-normal spaces called quasi $D\alpha$ -normal spaces by using $D\alpha$ -open sets due to Sayed and Khalil¹¹ in topological spaces and obtained several characterization and preservation theorems for quasi $D\alpha$ -normal spaces. We insure the existence of utility for new results using separation axioms in topological spaces which is separate on a known separation axioms in topological spaces.

Review of Literature

The notion of quasi normal space was introduced by Zaitsev¹³. Dontchev and Noiri² introduce the notion of πg -closed sets as a weak form of g -closed sets due to Levine [6]. By using πg -closed sets, Dontchev and Noiri [2] obtained a new characterization of quasi normal spaces. Sayed and Khalil [11] introduced the concept of $D\alpha$ -closed sets and discuss some of their basic properties. Recently, Reena et al. [8] introduced the concepts of quasi b^+ -normal spaces in topological spaces by using b^+ open sets in topological spaces and obtained some characterizations of such spaces.

Preliminaries

Definition

A subset A of a topological space X is called.

Regular Closed [12] If $A = Cl(Int(A))$.

Generalized Closed [4] (Briefly, g -closed) if $Cl(A) \subset U$ whenever $A \subset U$ and U is g -open in X .

πg -closed [2] If $Cl(A) \subset U$ whenever $A \subset U$ and U is π -open in X .

α -closed [7]

If $Cl(Int(Cl(A))) \subseteq A$. αg -closed [5]

If $\alpha-Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is in X .

$\pi g\alpha$ -closed [1] If $\alpha-Cl(A) \subset U$ whenever $A \subset U$ and U is π -open in X .

The finite union of regular open sets is said to be π -open. The complement of π -open set is said to be π -closed set. The complement of regular



M.C. Sharma

Associate Professor,
Deptt. of Mathematics,
N.R.E.C. College,
Khurja, U.P.

Poonam Sharma

Research Scholar,
Deptt. of Mathematics,
Mewar University,
Gangrar Chittorgarh,
Rajasthan

closed (resp. g -closed, π -open, πg -closed, α -closed, αg -closed, $\pi g \alpha$ -closed) set is said to be **regular open** (resp. **g -open, π -open, πg -open, α -open, αg -open, $\pi g \alpha$ -open**) sets. The intersection of all g -closed sets containing A is called the **g -closure of A** [3] and denoted by $Cl^*(A)$, and the **g -interior of A** [9] is the union of all g -open sets contained in A and is denoted by $Int^*(A)$.

Definition

A subset A of a topological space X is called, **$D\alpha$ -closed** [11] if $Cl^*(Int(Cl^*(A))) \subseteq A$.

$D\alpha g$ -closed If $Cl_\alpha^D(A) \subseteq U$ whenever $A \subseteq U$, and U is open in X .

$\pi g D\alpha$ -closed If $Cl_\alpha^D(A) \subset U$ whenever $A \subset U$ and U is π -open in X .

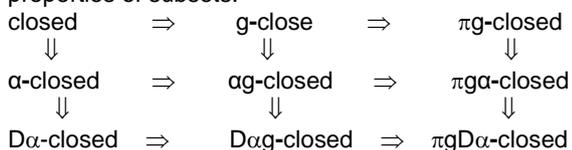
The complement of $D\alpha$ closed (resp. $D\alpha g$ -closed, $\pi g D\alpha$ -closed) sets is said to be **$D\alpha$ -open** (resp. **$D\alpha g$ -open, $\pi g D\alpha$ -open**). The intersection of all $D\alpha$ -closed subsets of X containing A (i.e. super sets of A) is called the **$D\alpha$ -closure of A** and is denoted by $Cl_\alpha^D(A)$. The union of all $D\alpha$ -open sets contained in A is called **$D\alpha$ -interior of A** and is denoted by $Int_\alpha^D(A)$. The family of all $D\alpha$ -open (resp. $D\alpha$ -closed) sets of a space X is denoted by **$D\alpha O(X)$** (resp. **$D\alpha C(X)$**).

Theorem [11].

Let X be a topological space. Then

1. Every α -closed subset of X is $D\alpha$ -closed.
2. Every g -open subset of X is $D\alpha$ -open.

We have the following implications for the properties of subsets.



Where none of the implications is reversible as can be seen from the following examples

Example

Let $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, \{a\}, \{c, d\}, \{a, c, d\}, \{d\}, \{a, d\}, X\}$. Then the set $A = \{a\}$ is $\pi g \alpha$ -closed set as well $\pi g D\alpha$ -closed set but not g -closed set in X .

Example

Let $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, d, c\}, \{a, b, d\}, \{a, b, c\}, X\}$. Then the set $A = \{a, b\}$ is $\pi g \alpha$ -closed set as well as $\pi g D\alpha$ -closed set but not αg -closed and not $D\alpha g$ -closed set in X . Since $A \subset \{a, b, c\}$ which is open by $Cl_\alpha^D \not\subset \{a, b, c\}$.

Example

Let $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, \{a\}, \{c, d\}, \{a, c, d\}, \{d\}, \{a, d\}, X\}$. Then the set $A = \{c\}$ is $\pi g \alpha$ -closed set as well as $\pi g D\alpha$ -closed set but not πg -closed set in X .

Theorem

1. Finite union of $\pi g D\alpha$ -closed sets are $\pi g D\alpha$ -closed.
2. Finite intersection of $\pi g D\alpha$ -closed need not be a $\pi g D\alpha$ -closed.

3. A countable union of $\pi g D\alpha$ -closed sets need not be a $\pi g D\alpha$ -closed.

Proof

1. Let A and B be $\pi g D\alpha$ -closed sets. Therefore $Cl_\alpha^D(A) \subset U$ and $Cl_\alpha^D(B) \subset U$ whenever $A \subset U, B \subset U$ and U is π -open. Let $A \cup B \subset U$ where U is π -open. Since $Cl_\alpha^D(A \cup B) \subset Cl_\alpha^D(A) \cup Cl_\alpha^D(B) \subset U$, we have $A \cup B$ is $\pi g D\alpha$ -closed.
2. Let $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}, X\}$. Let $A = \{a, b, c\}, B = \{a, b, d\}$. A and B are $\pi g D\alpha$ -closed sets. But $A \cap B = \{a, b\} \subset \{a, b\}$ which is π -open. $Cl_\alpha^D(A \cap B) \not\subset \{a, b\}$. Hence $A \cap B$ is not $\pi g D\alpha$ -closed.
3. Let R be the real line with the usual topology. Every singleton is $\pi g D\alpha$ -closed. But, $A = \{1/i : i = 2, 3, 4, \dots\}$ is not $\pi g D\alpha$ -closed. Since $A \subset (0, 1)$ which is π -open but $Cl_\alpha^D(A) \not\subset (0, 1)$.

Theorem

If A is $\pi g D\alpha$ -closed and $A \subset B \subset Cl_\alpha^D(A)$ then B is $\pi g D\alpha$ -closed.

Proof

Since A is $\pi g D\alpha$ -closed, $Cl_\alpha^D(A) \subset U$ whenever $A \subset U$ and U is π -open. Let $B \subset U$ and U be π -open. Since $B \subset Cl_\alpha^D(A), Cl_\alpha^D(B) \subset Cl_\alpha^D(A) \subset U$. Hence B is $\pi g D\alpha$ -closed.

Theorem

Let A be a $\pi g D\alpha$ -closed set in X . Then $Cl_\alpha^D(A) - A$ does not contain any non empty π -closed set.

Proof

Let F be a non empty π -closed set such that $F \subset Cl_\alpha^D(A) - A$. Then $F \subset Cl_\alpha^D(A) \cap (X - A) \subset X - A$ implies $A \subset X - F$ where $X - F$ is π -open. Therefore $Cl_\alpha^D(A) \subset X - F$ implies $F \subset (Cl_\alpha^D(A))^c$. Now $F \subset Cl_\alpha^D(A) \cap (Cl_\alpha^D(A))^c$ implies F is empty. Reverse implication does not hold.

Corollary

Let A be $\pi g D\alpha$ -closed. A is $D\alpha$ -closed iff $Cl_\alpha^D(A) - A$ is π -closed.

Proof. Let A be $D\alpha$ -closed set then $A = Cl_\alpha^D(A)$ implies $Cl_\alpha^D(A) - A = \emptyset$ which is π -closed.

Conversely if $Cl_\alpha^D(A) - A$ is π -closed then A is $D\alpha$ -closed.

Theorem

If A is π -open and $\pi g D\alpha$ -closed. Then A is $D\alpha$ -closed hence clopen.

Proof

Let A be regular open. Since A is $\pi g D\alpha$ -closed, $Cl_\alpha^D(A) \subset A$ implies A is $D\alpha$ -closed. Hence A is closed (Since every π -open, $D\alpha$ -closed set is closed). Therefore A is clopen.

Theorem

For a topological space X , the following are equivalent :

1. X is extremally disconnected.
2. Every subset of X is $\pi g D\alpha$ -closed.
3. The topology on X generated by $\pi g D\alpha$ -closed sets.

Proof

(a) \Rightarrow (b). Assume X is extremally disconnected. Let $A \subset U$, where U is π -open in X . Since U is π -open, it is the finite union of regular open sets and X is extremally disconnected, U is finite union of clopen sets and hence U is clopen. Therefore $Cl_\alpha^D(A) \subset Cl(A) \subset Cl(U) \subset U$ implies A is $\pi gD\alpha$ -closed.

(b) \Rightarrow (a). Let A be regular open set of X . Since A is $\pi gD\alpha$ -closed by **Theorem 2.11** A is clopen. Hence X is extremally disconnected.

(b) \Leftrightarrow (c) is obvious.

Lemma[11]

If A is a subset of X , then

1. $X - Cl_\alpha^D(A) = Int_\alpha^D(X - A)$.
2. $Cl_\alpha^D(X - A) = X - Int_\alpha^D(A)$.

Theorem

A subset A of a topological space X is $\pi gD\alpha$ -open if $F \subset Int_\alpha^D(A)$ whenever F is π -closed and $F \subset A$.

Proof

Let F be π -closed set such that $F \subset A$. Since $X - A$ is $\pi gD\alpha$ -closed and $X - A \subset X - F$ we have $F \subset Int_\alpha^D(A)$.

Conversely, Let $F \subset Int_\alpha^D(A)$ where F is π -closed and $F \subset A$. Since $F \subset A$ and $X - F$ is π -open, $Cl_\alpha^D(X - A) = X - Int_\alpha^D(A) \subset X - F$. Therefore A is $\pi gD\alpha$ -open.

Theorem

If, $Int_\alpha^D(A) \subset B \subset A$ and A is $\pi gD\alpha$ -open then B is $\pi gD\alpha$ -open.

Proof

Since, $Int_\alpha^D(A) \subset B \subset A$ using **Theorem 2.8**, $Cl_\alpha^D(X - A) \supset (X - B)$ implies B is $\pi gD\alpha$ -open.

Remark

For any $A \subset X$, $Int_\alpha^D(Cl_\alpha^D(A) - A) = \phi$.

Theorem

If $A \subset X$ is $\pi gD\alpha$ -closed then $Cl_\alpha^D(A) - A$ is $\pi gD\alpha$ -open.

Proof

Let A be $\pi gD\alpha$ -closed and F be a π -closed set such that $F \subset Cl_\alpha^D(A) - A$. By **Theorem 2.9**

$F = \phi$ implies $F \subset Int_\alpha^D(Cl_\alpha^D(A) - A)$. By

Theorem 2.14, $Cl_\alpha^D(A) - A$ is $\pi gD\alpha$ -open.

Converse of the above theorem is not true.

Example

Let $X = \{a, b, c\}$ and $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$. Let $A = \{b\}$. Then A is not $\pi gD\alpha$ -closed but $Cl_\alpha^D(A) - A = \{a, b\}$ $\pi gD\alpha$ -open.

Quasi $D\alpha$ -normal spaces

Definition

A topological space X is said to be **$D\alpha$ -normal** (resp. **quasi $D\alpha$ -normal**, **mildly $D\alpha$ -normal**) if for every pair of disjoint closed (resp. π -closed, regularly closed) subsets H, K of X , there exist disjoint $D\alpha$ -open sets U, V of X such that $H \subset U$ and $K \subset V$.

Example

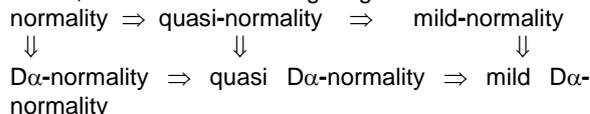
Let $X = \{a, b, c, d\}$ and $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, X\}$. The pair of disjoint closed subsets of X

are $A = \phi$ and $B = \{d\}$. Then $D\alpha$ -closed sets in X are $X, \phi, \{a\}, \{b\}, \{c\}, \{d\}, \{c, d\}, \{a, d\}, \{b, c\}, \{a, c\}, \{b, d\}, \{a, c, d\}, \{b, c, d\}$. Also $U = \{b\}$ and $V = \{c, d\}$ are $D\alpha$ -open sets such that $A \subset U$ and $B \subset V$. Hence X is $D\alpha$ -normal but it is not normal.

Example

Let $X = \{a, b, c, d\}$ and $\tau = \{\phi, \{a\}, \{c\}, \{a, c\}, \{b, d\}, \{a, b, d\}, \{b, c, d\}, X\}$. The pair of disjoint π -closed subsets of X are $A = \{a\}$ and $B = \{c\}$. Also $U = \{a\}$ and $V = \{b, c, d\}$ are open sets such that $A \subset U$ and $B \subset V$. Hence X is quasi-normal as well as quasi $D\alpha$ -normal because every open set is $D\alpha$ -open set.

By the definitions and examples stated above, we have the following diagram:



Theorem

For topological space X , the following are equivalent:

- a. X is quasi $D\alpha$ -normal.
- b. For any disjoint π -closed sets H and K , there exist disjoint $D\alpha g$ -open sets U and V such that $H \subset U$ and $K \subset V$.
- c. For any disjoint π -closed sets H and K , there exist disjoint $\pi gD\alpha$ -open sets U and V such that $H \subset U$ and $K \subset V$.
- d. For any π -closed set H and any π -open set V containing H , there exist a $D\alpha g$ -open set U of X such that $H \subset U \subset Cl_\alpha^D(U) \subset V$.
- e. For any π -closed set H and any π -open set V containing H , there exist a $\pi gD\alpha$ -open set U of X such that $H \subset U \subset Cl_\alpha^D(U) \subset V$.

Proof

(a) \Rightarrow (b), (b) \Rightarrow (c), (d) \Rightarrow (e), (c) \Rightarrow (d) and (e) \Rightarrow (a). (a) \Rightarrow (b).

Let X be quasi $D\alpha$ -normal. Let H, K be disjoint π -closed sets of X . By assumption, there exist disjoint $D\alpha$ -open sets U, V such that $H \subset U$ and $K \subset V$. Since every $D\alpha$ -open set is $D\alpha g$ -open, U, V are $D\alpha g$ -open sets such that $H \subset U$ and $K \subset V$.

(b) \Rightarrow (c). Let H, K be two disjoint π -closed sets. By assumption, there exists $D\alpha g$ -open sets U and V such that $H \subset U$ and $K \subset V$. Since $D\alpha g$ -open set is $\pi gD\alpha$ -open, U and V are $\pi gD\alpha$ -open sets such that $H \subset U$ and $K \subset V$.

(d) \Rightarrow (e). Let H be any π -closed set and V be any π -open set containing H . By assumption, there exist $D\alpha g$ -open set U of X such that $H \subset U \subset Cl_\alpha^D(U) \subset V$. Since every $D\alpha g$ -open set is $\pi gD\alpha$ -open, there exist $\pi gD\alpha$ -open sets U of X such that $H \subset U \subset Cl_\alpha^D(U) \subset V$.

(c) \Rightarrow (d). Let H be any π -closed set and V be any π -open set containing H . By assumption, there exist $\pi gD\alpha$ -open sets U and W such that $H \subset U$ and $X - V \subset W$. By **Theorem 2.14**, we get $X - V \subset Int_\alpha^D(W)$ and $Cl_\alpha^D(U) \cap Int_\alpha^D(W) = \phi$. Hence $H \subset U \subset Cl_\alpha^D(U) \subset X - Int_\alpha^D(W) \subset V$.

(e) \Rightarrow (a). Let H, K be any two disjoint π -closed set of X . Then $H \subset X - K$ and $X - K$ is π -open. By assumption, there exist $\pi g D\alpha$ -open set G of X such that $H \subset G \subset Cl_\alpha^D(G) \subset X - K$. Put $U = Int_\alpha^D(G), V = X - Cl_\alpha^D(G)$. Then U and V are disjoint $D\alpha$ -open sets of X such that $H \subset U$ and $K \subset V$.

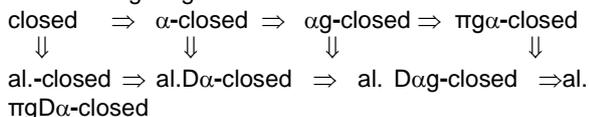
Some Functions

Definition

A function $f : X \rightarrow Y$ is said to be

1. Almost closed [10] (resp. almost $D\alpha$ -closed, almost $D\alpha g$ -closed) if $f(F)$ is closed (resp. $D\alpha$ -closed, $D\alpha g$ -closed) in Y for every $F \in RC(X)$.
2. $\pi g D\alpha$ -closed (resp. almost $\pi g D\alpha$ -closed) if for every closed set (resp. regularly closed) F of X , $f(F)$ is $\pi g D\alpha$ -closed in Y .
3. π -continuous [2] (resp. $\pi g\alpha$ -continuous[1], $\pi g D\alpha$ -continuous) if $f^{-1}(F)$ is π -closed (resp. $\pi g\alpha$ -closed, $\pi g D\alpha$ -closed) in X for every closed set F of Y .
4. Almost continuous [10] (resp. almost π -continuous [2], almost $\pi g\alpha$ -continuous[1], almost $\pi g D\alpha$ -continuous) if $f^{-1}(F)$ is closed (resp. π -closed, $\pi g\alpha$ -closed, $\pi g D\alpha$ -closed) in X for every regularly closed set F of Y .
5. Rc-preserving [6] if $f(F)$ is regularly closed in Y for every $F \in RC(X)$.

From the definitions stated above, we obtain the following diagram:



Where al. = almost

Moreover, by the following examples, we realize that none of the implications is reversible.

Example

$X = \{a, b, c, d\}, \tau = \{\phi, X, \{c\}, \{a, b, d\} \text{ and } \sigma = \{\phi, \{a\}, \{c, d\}, \{a, c, d\}, \{d\}, \{a, d\}, X\}$. Let $f : (X, \tau) \rightarrow (X, \sigma)$ be the identity function, then f is $\pi g\alpha$ -closed as well as $\pi g D\alpha$ -closed but not πg -closed. Since $A = \{c\}$ is not πg -closed in (X, σ) .

Example

Let $X = \{a, b, c, d\}, \tau = \{\phi, X, \{c\}, \{a, b, d\}, \{b, d\}, \{b, c, d\}, X\}$ and $\sigma = \{\phi, X, \{a\}, \{c, d\}, \{a, c, d\}, \{d\}, \{a, d\}\}$. Let $f : (X, \tau) \rightarrow (X, \sigma)$ be the identity function. Then f is almost $\pi g\alpha$ -closed as well as almost $\pi g D\alpha$ -closed but not $\pi g D\alpha$ -closed. Since $A = \{a\}$ is not $\pi g D\alpha$ -closed

Theorem

If $f : X \rightarrow Y$ is an almost π -continuous and $\pi g D\alpha$ -closed function, then $f(A)$ is $\pi g D\alpha$ -closed in Y for every $\pi g D\alpha$ -closed set A of X .

Proof

Let A be any $\pi g D\alpha$ -closed set A of X and V be any π -open set of Y containing $f(A)$. Since f is almost π -continuous, $f^{-1}(V)$ is π -open in X and $A \subset f^{-1}(V)$. Therefore $Cl_\alpha^D(A) \subset f^{-1}(V)$ and hence $f(Cl_\alpha^D(A)) \subset V$. Since f is $\pi g D\alpha$ -closed, $f(Cl_\alpha^D(A))$ is

$\pi g D\alpha$ -closed in Y . And hence we obtain $Cl_\alpha^D(f(A)) \subset Cl_\alpha^D(f(Cl_\alpha^D(A))) \subset V$. Hence $f(A)$ is $\pi g D\alpha$ -closed in Y .

Theorem

A surjection $f : X \rightarrow Y$ is almost $\pi g D\alpha$ -closed if and only if for each subset S of Y and each $U \in RO(X)$ containing $f^{-1}(S)$ there exists a $\pi g D\alpha$ -open set V of Y such that $S \subset V$ and $f^{-1}(V) \subset U$.

Proof

Necessity, suppose that f is almost $\pi g D\alpha$ -closed. Let S be a subset of Y and $U \in RO(X)$ containing $f^{-1}(S)$. If $V = Y - f(X - U)$, then V is a $\pi g D\alpha$ -open set of Y such that $S \subset V$ and $f^{-1}(V) \subset U$.

Sufficiency, Let F be any regular closed set of X . Then $f^{-1}(Y - f(F)) \subset X - F$ and $X - F \in RO(X)$. There exists $\pi g D\alpha$ -open set V of Y such that $Y - f(F) \subset V$ and $f^{-1}(V) \subset X - F$. Therefore, we have $f(F) \supset Y - V$ and $F \subset X - f^{-1}(V) \subset f^{-1}(Y - V)$. Hence we obtain $f(F) = Y - V$ and $f(F)$ is $\pi g D\alpha$ -closed in Y which shows that f is almost $\pi g D\alpha$ -closed.

Preservation Theorem

Theorem

If $f : X \rightarrow Y$ is an almost $\pi g D\alpha$ -continuous, rc-preserving injection and Y is quasi $D\alpha$ -normal then X is quasi $D\alpha$ -normal.

Proof

Let A and B be any disjoint π -closed sets of X . Since f is an rc-preserving injection, $f(A)$ and $f(B)$ are disjoint π -closed sets of Y . Since Y is quasi $D\alpha$ -normal, there exist disjoint $D\alpha$ -open sets U and V of Y such that $f(A) \subset U$ and $f(B) \subset V$.

Now if $G = Int(Cl(U))$ and $H = Int(Cl(V))$. Then G and H are regularly open sets such that $f(A) \subset G$ and $f(B) \subset H$. Since f is almost $\pi g D\alpha$ -continuous, $f^{-1}(G)$ and $f^{-1}(H)$ are disjoint $\pi g D\alpha$ -open sets containing A and B which shows that X is quasi $D\alpha$ -normal.

Theorem

If $f : X \rightarrow Y$ is π -continuous, almost $D\alpha$ -closed surjection and X is quasi $D\alpha$ -normal space then Y is $D\alpha$ -normal.

Proof

Let A and B be any two disjoint closed sets of Y . Then $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint π -closed sets of X . Since X is quasi $D\alpha$ -normal, there exist disjoint $D\alpha$ -open sets of U and V such that $f^{-1}(A) \subset U$ and $f^{-1}(B) \subset V$. Let $G = Int(Cl(U))$ and $H = Int(Cl(V))$. Then G and H are disjoint regularly open sets of X such that $f^{-1}(A) \subset G$ and $f^{-1}(B) \subset H$. Set $K = Y - f(X - G)$ and $L = Y - f(X - H)$. Then K and L are $D\alpha$ -open sets of Y such that $A \subset K, B \subset L, f^{-1}(K) \subset G, f^{-1}(L) \subset H$. Since G and H are disjoint, K and L are disjoint.

Since K and L are $D\alpha$ -open and we obtain $A \subset Int_\alpha^D(K), B \subset Int_\alpha^D(L)$ and $Int_\alpha^D(K) \cap Int_\alpha^D(L) = \phi$. Therefore Y is $D\alpha$ -normal.

Theorem

Let $f : X \rightarrow Y$ be an almost π -continuous and almost $\pi g D_\alpha$ -closed surjection. If X is quasi D_α -normal space then Y is quasi D_α -normal.

Proof. Let A and B be any disjoint π -closed sets of Y . Since f is almost π -continuous, $f^{-1}(A)$, $f^{-1}(B)$ are disjoint closed subsets of X . Since X is quasi D_α -normal, there exist disjoint D_α -open sets U and V of X such that $f^{-1}(A) \subset U$ and $f^{-1}(B) \subset V$.

Let $G = \text{Int}(\text{Cl}(U))$ and $H = \text{Int}(\text{Cl}(V))$. Then G and H are disjoint regularly open sets of X such that $f^{-1}(A) \subset G$ and $f^{-1}(B) \subset H$. By Theorem 4.5, there exist $\pi g D_\alpha$ -open sets K and L of Y such that $A \subset K$, $B \subset L$, $f^{-1}(K) \subset G$ and $f^{-1}(L) \subset H$. Since G and H are disjoint, so are K and L by Theorem 2.14, $A \subset \text{Int}_\alpha^D(K)$, $B \subset \text{Int}_\alpha^D(L)$ and $\text{Int}_\alpha^D(K) \cap \text{Int}_\alpha^D(L) = \emptyset$. Therefore Y is quasi D_α -normal.

Corollary

If $f : X \rightarrow Y$ is almost continuous and almost closed surjection and X is a normal space, then Y is quasi D_α -normal.

Proof

Since every almost closed function is almost $\pi g D_\alpha$ -closed so Y is quasi D_α -normal.

Conclusion

The notion of quasi D_α -normal in topological spaces has been generalized and obtain characterizations and preservation theorems of quasi D_α -normal.

References

1. Arockiarani and C. Janaki, $\pi g \alpha$ -closed sets and quasi α -normal spaces, *Acta Ciencia Indica*, Vol. XXXIII M. No. 2, (2007), 657-666.

2. J. Dontchev and T. Noiri, Quasi-normal spaces and πg -closed sets, *Acta Math. Hungar.* 89(3)(2000), 211-219.
3. Dunham, W., A new closure operator for non- T_1 topologies, *Kyungpook Math. J.* 22(1982), 55 - 60.
4. N. Levine, Generalized closed sets in topology, *Rend. Circ. Mat. Palermo* 19(1970),89-96.
5. H. Maki, R. Devi and Balachandran K., Generalized α -closed sets in topology, *Bull. Fukuoka Univ. ed. Part III* 42 (1993), 13-21.
6. T. Noiri, Mildly - normal spaces and some functions. *Kyungpook Math. J.* 36(1996),183 - 190.
7. O Njastad, On some class of nearly open sets, *Pacific. J. Math.*, 15(1965), 961-970.
8. S.Reena, F.Nirmala Irudayam, A new weaker form of $\pi g b$ - continuity, *International J. of Innovative Research in Sci., Engineering and Tech.* Vol. 5 , No.5 (2016) ,8676-8682.
9. Robert, A., Missier S. P., On semi*-closed sets, *Asian J. Engineering Math.* 4(2012),173-176.
10. M. K. Singal and A. R. Singal, Almost continuous mappings, *Yokohama Math. J.* 16(1968), 63-73.
11. O.R. Sayed, A.M. Khalil, Some applications of D_α -closed sets in topological spaces, *Egyptian J. of Basic and Applied Sci.* (2015),doi: 10.1016/j.2015.07.005,1-9.
12. M.H. Stone, Applications of the theory of Boolean rings to general topology, *Trans. Amer. Math. Soc.* 41 (1937), 375-381.
13. Zaitsev V., On certain classes of topological spaces and their biocompactifications, *Dokl Akad Nauk SSSR* 178(1968), 778-779.