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E: ISSN NO.: 2349-980X **Shrinkhla Ek Shodhparak Vaicharik Patrika**

A Two Unit Standby System with Different Operative Modes and Different Repair Policies

Abstract

The paper deals with a two non- identical unit cold standby system model. One unit is named as priority (p) unit and other as ordinary (o) unit. Priority unit has two operative modes- normal and quasi- normal whereas ordinary unit has only one operative mode say normal. When p-unit fails first time it enters into minor repair and after minor repair, the unit does not become as good as new and operates with increased failure rate. When p- unit again fails, it enters into complete overhauling and after this action unit works as good as new. The ordinary unit upon failure enters into minor repair and after repair it becomes as good as new. A single repairman is always available with the system for both types of repair.

Keywords: Regenerative point, Reliability, MTSF, Busy period of repairman, Net Expected Profit.

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Introduction

Repairable parallel systems have attracted the attention of several authors in the field of reliability theory. Nakagawa and Osaki [19], Subramaniam and Ravichandran[20], Jaiswal and Krishna[14], have studied two unit standby system model under following assumptions :

1. Unit behaves like a new one after each repair i.e. its failure time distribution remains unaltered as a consequence of repair.
2. Repair time distribution remains the same after each failure.

In fact these assumptions may not be imposed on every system as the repair characteristics vary from system to system.

Using regenerative point technique, the following economic measures of the system effectiveness are obtained:

1. Reliability and mean time to system failure (MTSF).
2. Pointwise and steady-state availabilities of the system and expected up time of the system during time (0, t).
3. Busy period analysis of the repairman in both types (minor and major) of repair.
4. Net expected profit earned by the system during time (0, t) and in steady- state.

Aim of the paper

The purpose of the present paper is to analyze a two non-identical unit cold standby system model with two modes of a unit (N, F) out of which N and F are the normal and failure mode of the system. This paper is based on the concept of two types of repair depending upon the operative modes of the priority unit have been introduced. The priority unit when fails for the first time goes into minor repair to restore it to operative mode having increased failure rate. When the priority unit fails second time, a major repair or complete overahaul is performed, so as to bring the unit as good as new.

Model Description and assumptions

1. The system comprises of two non-identical units. One unit is named as priority (p) unit and other as ordinary (o) unit.
2. Priority unit get preference both for operation and repair.
3. A single repairman is available with the system to repair a p and o units.
4. When p- unit fails it enters into minor repair. After minor repair the unit does not become as good as new and operates with increased failure rate.

5. When p-unit fails second time it enters into major repair and a major repaired p-unit works as good as new.
6. The o-unit has only one operative mode and upon failure, it always goes to minor repair facility and after repair unit becomes as good as new.
7. Switching device is to transfer the standby unit into operation and to change the operation of the system from ordinary unit to priority unit is always perfect and instantaneous.
8. If during the repair of o- unit, p-unit fail the minor repair of p- unit starts discontinuing the repair of o-unit, The residual repair of o- unit is pre-emptive repeat type i.e. the time already spent in the repair of o-unit goes to waste.
9. All the repair time distributions are taken general with different cdf and the failure time distributions are taken as exponential with different parameters.

Review of Literature

A TWO – UNIT STANDBY SYSTEM WITH TWO OPERATIVE MODES OF THE UNITS AND PREPARATION TIME FOR REPAIR

This paper based on a stochastic behavior of a two non- identical unit cold standby system model assuming three mode - normal, partially failure and total failure of the units. A totally failed unit needs some preparation work before going into repair and after completion of preparation, the unit is sent for repair. A partially failed unit may operate with reduced efficiency while it is undergoing repair. A single repairman plays the triple role – the repair of partially failed unit, the preparation of repair of totally failed unit and repair of totally failed unit. The repair discipline is FCFS in respect of above three jobs.

Notations and states of the system

(a) Notations

α_1 : Constant failure rate of p- unit in N-mode.

α_2 (>α_1) : Constant failure rate of p- unit in $N^{'}$ -mode.

β: Constant failure rate of o- unit.

$G_1(.)$: cdf of minor repair time of p-unit and o-unit.

G_2 (.) : cdf of major repair time of p-unit.

(b) Symbols used for the states of the system

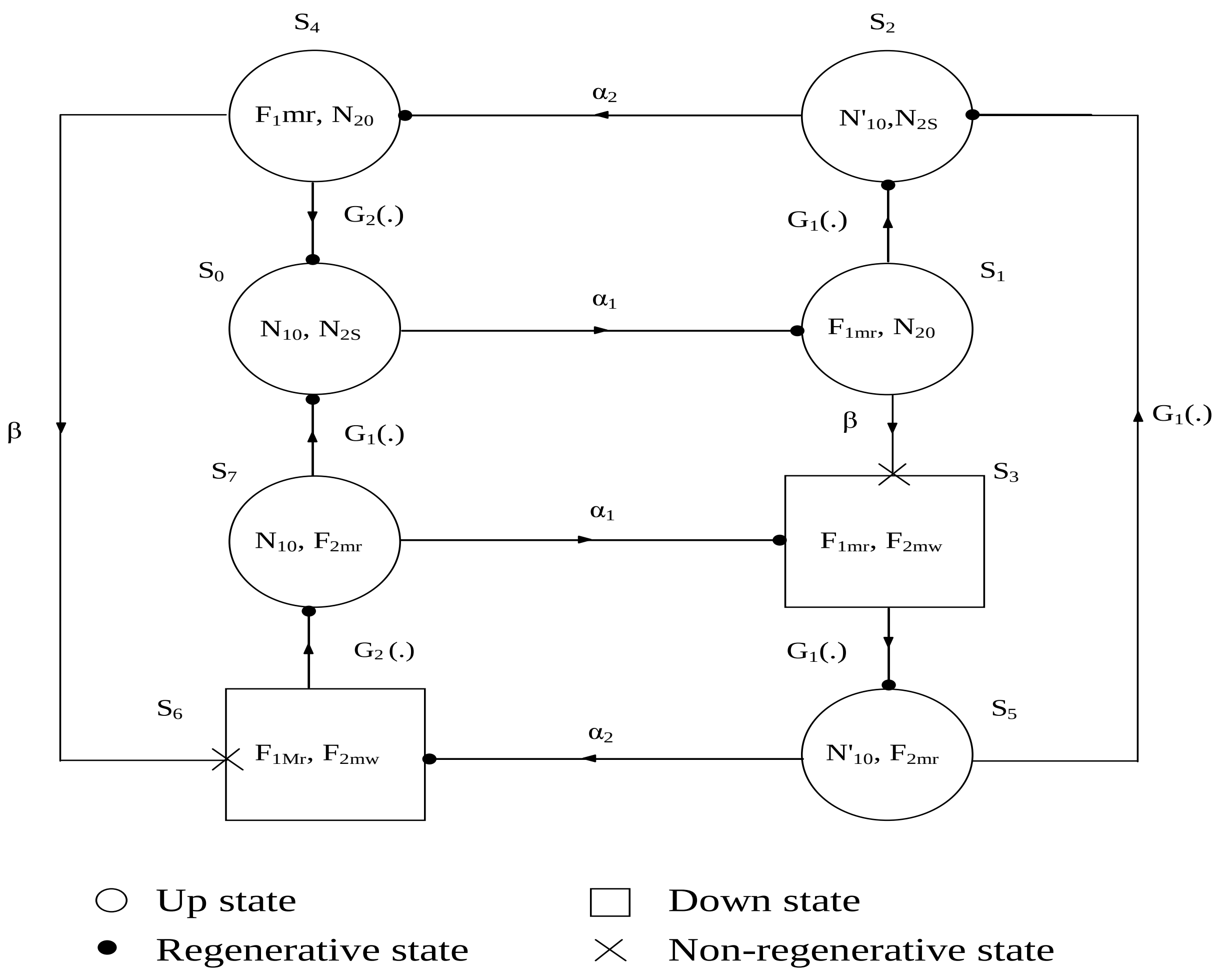
$N_{1o}, N_{1o}^{'}$: p-unit is in (N) normal mode/($N^{'}$) quasi-normal mode.

N_{2o}, N_{2S}: o - unit is operative in N-mode and operative/standby.

F_{1mr}, F_{1Mr} : p - unit is in failure(F) mode and under minor/major repair.

F_{2mr}, F_{2mw} : o-unit is in failure(F) mode and under minor repair/waiting for minor repair.

TRANSITION DIAGRAM



The transition diagram of the system model is shown in the figure where we observe that the epochs of entrance from S_1 to S_3 and from S_4 to S_6 are non-regenerative whereas all other entrance points are regenerative.

Transition Probabilities and Sojourn Times

(a) The steady- state unconditional and conditional transition probabilities can be obtained as follows

$p_{01} = 1$
$p_{12} = \tilde{G}_1(\beta)$
$p_{15}^{(3)} = 1 - \tilde{G}_1(\beta)$
$p_{24} = 1$
$p_{35} = 1$
$p_{40} = \tilde{G}_2(\beta)$
$p_{47}^{(6)} = 1 - \tilde{G}_2(\beta)$
$p_{52} = \tilde{G}_1(\alpha_2)$
$p_{56} = 1 - \tilde{G}_1(\alpha_2)$
$p_{67} = 1$
$p_{70} = \tilde{G}_1(\alpha_1)$
$p_{73} = 1 - \tilde{G}_1(\alpha_1)$

It can easily verified that

$p_{12} + p_{15}^{(3)} = 1 \qquad p_{40} + p_{47}^{(6)} = 1$
$p_{52} + p_{56} = 1 \qquad p_{70} + p_{73} = 1$

(b) The mean sojourn time in various states as follows

$\Psi_0 = \int P(T_0 > t)dt = 1/\alpha_1$
$\Psi_1 = [1 - \tilde{G}_1(\beta)]/\beta$

$$\Psi_2 = 1/\alpha_2$$

$$\Psi_3 = \int \bar{G}_1(t)dt = n_1(\text{mean minor repair time of p} - \text{unit})$$

$\Psi_4 = [1 - \tilde{G}_2(\beta)]/\beta$
$\Psi_5 = [1 - \tilde{G}_1(\alpha_2)]/\alpha_2$

$$\Psi_6 = \int \bar{G}_2(t)dt = n_2(\text{mean major repair time of p} - \text{unit})$$

$\Psi_7 = [1 - \tilde{G}_1(\alpha_1)]/\alpha_1$

5. Analysis of Characteristics

(a) Reliability and MTSF

To determine $R_i(t)$, the reliability of the system when system initially starts from state regenerative S_i, we assume that failed states S_3 and S_6 of the system as absorbing. Using simple probabilistic arguments in regenerative point technique, the value of $R_0(t)$ in terms of its Laplace transform is

$$R_0^*(s) = \frac{Z_0^* + q_{01}^*[Z_1^* + q_{12}^*(Z_2^* + q_{24}^* Z_4^*)]}{1 - q_{01}^* q_{12}^* q_{24}^* q_{40}^*}$$

where Z_0^*, Z_1^*, Z_2^* and Z_4^* are the L.T. of

$Z_0(t) = e^{-\alpha_1 t}, \qquad Z_1(t) = e^{-\beta t}\overline{G_1}(t)$
$Z_2(t) = e^{-\alpha_2 t}, \qquad Z_4(t) = e^{-\beta t} G_2(t)$

The mean time to system failure is given by

$$E(T_0) = \lim_{s \to 0} R_0^*(s)$$

$$E(T_0) = \frac{\Psi_0 + [\Psi_1 + p_{12}(\Psi_2 + \Psi_4)]}{1 - p_{12} p_{40}}$$

(b) Availability Analysis

Let $A_i^{pN}(t)$, $A_i^{pN'}(t)$ and $A_i^{O}(t)$ be the probability that the system is up at epoch t due to priority unit in N mode, N' mode and due to the operation of o-unit, when initially system starts operation from state $S_i \in$ E. Using the regenerative point technique and the tools of Laplace transforms, one can obtain the values of $A_0^{pN}(t)$, $A_0^{pN'}(t)$ and $A_0^{O}(t)$ in terms of their Laplace Transformations i.e. $A_0^{pN*}(s)$, $A_0^{pN'}(s)$ and $A_0^{O*}(s)$.

The steady-state availabilities of the system due to p-unit in N mode, p- unit in N' mode and ordinary unit are given by

$A_0^{pN} = N_1/D_1, \quad A_0^{pN'} = N_2/D_1$ and $A_0^{O} = N_3/D_1$

where,

$N_1 = (p_{40}p_{52} + p_{47}^{(6)}p_{70} + p_{40}p_{56}p_{70})\Psi_0 + (p_{47}^{(6)} + p_{40}p_{56}p_{15}^{(3)})\Psi_7$

$N_2 = (1 - p_{56} + p_{12}p_{56}p_{70})\Psi_2 + (1 - p_{12}p_{40} - p_{12}p_{70} + p_{12}p_{40}p_{70})\Psi_5$

$N_3 = (p_{40}p_{52} + p_{47}^{(6)}p_{70} + p_{40}p_{56}p_{70})\Psi_1 + (1 - p_{56} + p_{12}p_{56}p_{70})\Psi_4$

$D_1 = \left(p_{40}p_{52} + p_{47}^{(6)}p_{70} + p_{40}p_{56}p_{70}\right)\Psi_0 + (1 - p_{56} + p12p56p70\Psi2 + (1 - p_{12}p_{40} - p_{12}p_{70} + p_{12}p_{40}p_{70})\Psi_5 + \left(p_{47}^{(6)} + p40p153p56\Psi7 + (n1+n2)(1-p12p40p56+p12p40p56p70)\right)$

(c) Busy Period Analysis (10)

In Minor repair (1-4)

Let $B_i^m(t)$ be the probability that the repair facility is busy in minor repair, when initially system starts functioning from state $S_i \in$ E. Using the regenerative point technique and the tools of Laplace transform, one can obtain the value of $B_i^m(t)$ in terms of their Laplace transform i.e. $B_i^{m*}(s)$. In the long run the probability that the repair facility will be busy in minor repair is given by

$$B_0^m = N_4/D_1 \qquad (11)$$

Where,

$N_4 = (p_{40}p_{52} + p_{47}^{(6)}p_{70} + p_{40}p_{56}p_{70})\Psi_1 + (1 - p_{40} + p_{40}p_{56} - p_{70} + p_{40}p_{70} - p_{40}p_{56}p_{70} - p_{12}p_{40}p_{56} + p_{12}p_{40}p_{56}p_{70})\Psi_3 + (1 - p_{12}p_{40} - p_{12}p_{70} + p_{12}p_{40}p_{70})\Psi_5 + (p_{47}^{(6)} + p_{40}p_{15}^{(3)}p_{56})\Psi_7$

The value of D_1 is same as given in (b).

In Major repair

Let $B_0^M(t)$ be the probability that the repair facility is busy in major repair, when initially system starts functioning from state $S_i \in$ E. In the long run the probability that the repair facility will be busy in major repair is given by

(5) $\qquad B_0^M = N_5/D_1 \qquad$ 12)

Where,

$N_5 = (1 - p_{56} + p_{12}p_{56}p_{70})\Psi_4 + (p_{56} - p_{12}p_{40}p_{56} - p_{12}p_{70}p_{56} + p_{12}p_{40}p_{56}p_{70})\Psi_6$

The value of D_1 is same as given in (b).

(d) Cost Benefit Analysis

Let K_0, K_1 and K_2 be the per unit up time revenues by the system due to (6) operation of p- unit in N mode, p- unit in N' mode and o- unit in N mode respectively. K_3 and K_4 be the repair cost per unit itme when a unit is under minor and major repair respectively. Then the net expected profit incurred by the system during time interval (0, t) is given by

$$P(t) = K_0\mu_{up}^{pN}(t) + K_1\mu_{up}^{pN'}(t) + K_2\mu_{up}^{O}(t) - K_3\mu_b^m(t) - K_4\mu_b^M(t) \qquad (13)$$

The expected profit per- unit time in steady-state is

$$P = K_0A_0^{pN} + K_1A_0^{pN'} + K_2A_0^{o} - K_3B_0^{m} - K_4B_0^{M} \quad (14)$$

Conclusion

In this standby system we see that there are two types of operative modes and repair policies. So there will be always a benefit of customers because after minor repair the p-unit operates with increased failure rate and when p- unit fails second time it enters into major repair and after it unit becomes as good as new.

References

1. Singh, R.P., Taneja, G. and Siwach, B.S. “Two unit warm standby system with accident and various types of repair”. Journal of decision and mathematical sciences, 5-6 (1-3), pp75-86, 2000.
2. Pradeep Chaudhary and Dharmendra Kumar, “Stochastic analysis of a two unit cold standby system with different operative modes and different repair policies”. International Journal J. Agriculture Statistical Sciences, vol. 3, No. 2, pp. 387-394, (2007).
3. Amir Azaron, Hideki Katagiri, Kosuke Kato, MassatoshiSakawa, “Reliability evaluation and optimization of dissimilar-component cold standby redundant system”. Journal of the operations Research Society of Japan, vol. 48 No. 1, 71-8,(2005).
4. Mogha, A.K. and A.K. Gupta, “A two priority unit warm stand by system model with preparation for repair”Aligarh J of statistics, 22, 73-90(2002).
5. Vikas Sharma, J.P. Singh Joorel, AnkushBharti and RakeshChib, Stochastic behavior of a two unit system with partial failure and fault detection. IJRCIM, Vol. 2, No. 06, 90-9(2012).
6. Gupta, R. and Mumtaj,S.Z. (1995). A cold standby system with repair and replacement policies and correlated failure and repair times, J, of Quality and Maintenance Engineering, Vol. 1, p. 55-66.
7. Agnihotri, R.K. and Rastogi, S.K. (1996). Two unit identical systems with priority based repair and inspection, Microelectron. Reliab., Vol. 36, p.279-282.
8. Gupta,R. and Goel, L. R. (1990). Cost benefit analysis of two unit parallel sytem with administrative delay in repair, Int. J. of System Science, Vol. 21,p. 1369-1379.
9. Dhillon B.S. and N. Yang, “Reliability and availability of warm standby system with common cause failure and human error”. Micro electron Reliab., 23(2),337-576(1992).
10. Gupta R., P.K. Tyagi and Ram Kishan“ A two unit system with correlated failures and repairs and random appearance and disappearance of repairman”. Int. J. of system svienc., 27(6), pp. 561-566(1996).