A Meta-Heuristic Approach for Solving Economical Ordered Quantity (EOQ) Problem

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Abstract

Presently many inventory models have been developed to find the optimum value of Economical Ordered Quantity (EOQ) for deteriorating items. Through the development of a mathematical model, the optimal order size is derived using traditional mathematical and statistical calculation methods which required complex mathematical expressions to be solved analytically. This paper is intended to view the applicability of meta-heuristic algorithms to solve this problem since the meta-heuristic algorithms may provide a sufficiently good solution to an optimization problem, especially with incomplete or imperfect information and with lower computational complexity. To verify the proposed approach in this paper the EOQ inventory mathematical model for deteriorating items with exponentially decreasing demand is accepted and the PSO (Particle Swarm Optimization) algorithm is selected as meta-heuristic algorithm. The model also considered shortages and partially backlogging. The particular model is selected because a lots of literature is already available to cross validate the proposed approach.

Keywords: PSO (Particle Swarm Optimization), EOQ, Inventory, Deteriorating Items.

Introduction

In recent years more attention is paid to the supply chain systems, because of rising costs, globalization of resources, globalization of economy, increased verities of products and requirements of lower response time. Because of large variability in the influencing conditions, basic assumptions of the EOQ model requires frequent changes in accordance with the inventory. Recently the attention to another important aspect known as deteriorating properties of items has been largely studies for the accurate modeling of supply chain systems.

Since many other important variables may be considered to more realistic model formation, many researchers have already proposed the different kinds of EOQ models for different items. Like no-shortage inventory model with constant speed decay, inventory model with Weibull-distributed deteriorating rate, inventory model with discount and partial backordering etc.

Finally in this, an EOQ inventory model with deteriorating items is taken [1], which considered that the demand function is exponentially decreasing and the backlogging rate is inversely proportional to the waiting time for the next replenishment. The objective of the problem is to minimize the total relevant cost by simultaneously optimizing the shortage point and the length of cycle.

Literature Review

In recent years, a lots of research have been performed on EOQ models for deteriorating items. Liang Yuh Ouyangetal.(1) presented an EOQ inventory mathematical model for deteriorating items with exponentially decreasing demand. Their model also handles the shortages and variable rate partial back ordering which dependents on the waiting time for the next replenishment. Kai-Wayne Chuang et al.(2) studied pricing strategies in marketing, with objective to find the optimal inventory and pricing strategies for maximizing then et present value of total profit over the infinite horizon. The studied two variants of models: one without considering shortage, and the other with shortage. Jonas C.P. Yu (4) developed a deteriorating inventory system with only one supplier and one buyer. The system considers the collaboration and trade credit between supplier and buyer. The objective is to maximize the total profit of the whole system when shortage is completely backordered**.** The literatures also discuss the negotiation mechanism between supplier and buyer in case of . shortages and payment delay. Lianxia Zhao7() studied an inventory model with trapezoidal type demand rate and partially backlogging for Weibull-distributed deterioration item sand derived an optimal inventory replenishment policy. Kuo-Lung Hou et al. [10] presents an inventory model for deteriorating items considering the stock-dependent selling rate under inflation and time value of money over a finite planning horizon. The model allows shortages and partially backlogging at exponential rate. Ching-Fang Lee et al. Al [14], considered system dynamics to propose a new order system for integrated inventory model of supply chain for deteriorating items among a supplier a producer and a buyer. The system covers the dynamics of complex relation due to time evolution.

Mathematical Modeling Of System And Problem Formulation

The mathematical model in this paper is rendered from reference [1] with following notation and assumptions.

Notation:

c_1:Holding cost, (\$/per unit)/per unit time.
c_2:Cost of the inventory item, \$/per unit.
c_3:Ordering cost of inventory, \$/per order.
c_4:Shortage cost, (\$/per unit)/per unit time.
c_5:Opportunity cost due to lost sales, \$/per unit.
t_1 :Time at which shortages start.
T :Length of each ordering cycle.
W :The maximum inventory level for each ordering cycle.
S :The maximum amount of demand backlogged for each ordering cycle.
Q :The order quantity for each ordering cycle.
Inv(t): The inventory level at time t.

Assumptions

1. The inventory system involves only one item and the planning horizon is infinite.
2. The replenishment occurs instantaneously at an infinite rate.
3. The deteriorating rate, $\theta\ (0 < \theta < 1)$, is constant and there is no replacement or repair of deteriorated units during the period under consideration.
4. The demand rate $R(t)$, is known and decreases exponentially.
$$R(t) = \begin{cases} Ae^{-\lambda t}, I(t) > 0 \\ D\ , I(t) \leq 0 \end{cases}$$
Where $A\ (> 0)$is initial demand and $\lambda\ (0 < \lambda < \theta)$is a constant governing the decreasing rate of the demand.
5. During the shortage period, the backlogging rate is variable and is dependent on the length of the waiting time for the next replenishment. The longer the waiting time is, the smaller the backlogging rate would be. Hence, the proportion of customers who would like to accept back logging at time t is decreasing with the waiting time$(T - t)$waiting for the next replenishment. To take care of this situation we have defined the backlogging rate to be $\frac{1}{1+\delta(T-t)}$when inventory is

negative. The backlogging parameter δ is a positive constant $t_1 < t < T$.

Model Formulation

Here, the replenishment policy of a deteriorating item with partial backlogging is considered. The objective of the inventory problem is to determine the optimal order quantity and the length of ordering cycle so as to keep the total relevant cost as low as possible. The behavior of inventory system at any time is depicted in Figure 1.

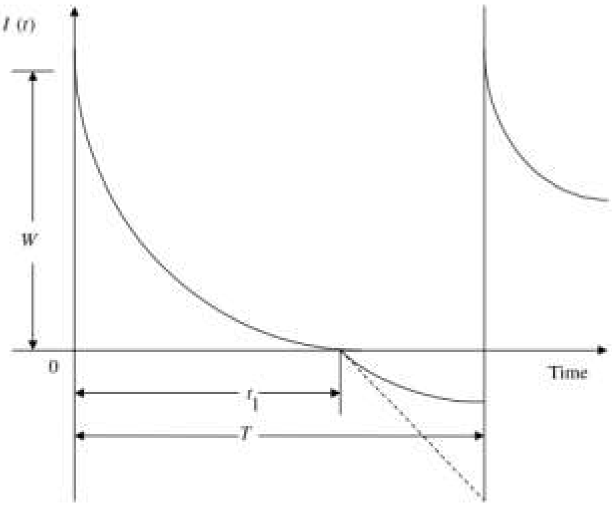

Figure 1: Inventory level$Inv(t\ ) vs. t\ (time)$.

Replenishment is made at time$t = 0$and the inventory level is at its maximum W.Due to both the market demand and deterioration of the item, the inventory level decreases during the period $[0, t_1]$ and ultimately falls to zero at $t = t_1$.Thereafter, shortages are allowed to occur during the time interval $[t_1, T]$ and all of the demand during the period $[t_1, T]$ is partially backlogged.

As described above, the inventory level decreases owing to demand rate as well as deterioration during inventor interval $[0, t_1]$. Hence the differential equation representing the
inventory status is given by

$$\frac{dInv(t)}{dt} + \theta Inv(t) = -Ae^{-\lambda t}, 0 \leq t \leq t_1 \ldots\ldots.. \quad (1)$$

with the boundary condition$I(0) = W$. The solution of equation (1) is

$$Inv(t) = \frac{Ae^{-\lambda t}}{\theta - \lambda}\left[e^{(\theta-\lambda)(t_1-1)} - 1\right], 0 \leq t \leq t_1 \ldots\ldots\ldots (2)$$

So the maximum inventory level for each cycle can be obtained as

$$W = Inv(0) = \frac{A}{\theta - \lambda}\left[e_1^{(\theta-\lambda)t} - 1\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(3)$$

During the shortage interval $[t_1, T]$, the demand at time t is partly backlogged at the fraction$\frac{1}{1+\delta(T-t)}$Thus, the differential equation governing the amount of demand backlogged is as below.

$$\frac{dInv(t)}{dt} = \frac{D}{1 + \delta(T - t)}, t_1 < t \leq T \ldots\ldots\ldots\ldots\ldots. \quad (4)$$

with the boundary condition $I(t_1) = 0$. The solution of equation (4) can be given by

$$Inv(t) = \frac{D}{\delta}\{\ln[1 + \delta(T - t)] - \ln[1 + \delta(T - t_1)]\}, t_1 \leq t \leq T \ldots\ldots.. \quad (5)$$

Let $t = T$ in (5), we obtain the maximum amount of demand backlogged per cycle as follows:

$$S = -Inv(T) = \frac{D}{\delta}\ln[1 + \delta(T - t_1)] \ldots\ldots\ldots\ldots \quad (6)$$

Hence, the order quantity per cycle is given by

$$Q = W + S = \frac{A}{\theta - \lambda}\left[e^{(\theta-\lambda)t_1} - 1\right] + \frac{D}{\lambda}\ln[1 + \delta(T - t_1)] \ldots\ldots\ldots\ldots (7)$$

The inventory holding cost per cycle is

$$HC = \int_0^{t_1} c_1 Inv(t)dt = \frac{c_1 A}{\theta(\theta - \lambda)} e^{-\lambda t_1}\left[e^{\theta t_1} - 1 - \frac{\theta}{\lambda}\left(e^{\lambda t_1} - 1\right)\right] \ldots\ldots\ldots \quad (8)$$

The deterioration cost per cycle is

$$DC = c_2[W - \int_0^{t_1} R(t)dt] = c_2[W - \int_0^{t_1} Ae^{-\lambda t}] = c_2 A\left\{\frac{1}{\theta - \lambda}\left(e^{(\theta-\lambda)t_1} - 1\right) - \frac{1}{\lambda}\left(1 - e^{-\lambda t_1}\right)\right\} \ldots\ldots (9)$$

The shortage cost per cycle is

$$SC = c_4\left[-\int_{t_1}^{T} I(t)dt\right] = c_4 D\left\{\frac{T - t_1}{\delta} - \frac{1}{\delta^2}\ln[1 + \delta(T - t_1)]\right\} \ldots\ldots\ldots (10)$$

The opportunity cost due to lost sales per cycle is

$$BC = c_5\int_{t_1}^{T}\left[1 - \frac{1}{1 + \delta(T - t)}\right]D\,dt = c_5 D\left\{(T - t_1) - \frac{1}{\delta}\ln[1 + \delta(T - t_1)]\right\} \ldots \quad \ldots \quad ..(11)$$

Therefore, the average total cost per unit time per cycle is

$$TVC \equiv TVC(t_1, T)$$

= (holding cost + deterioration cost+ ordering cost+ shortage cost+ opportunity cost due to lost sales)/ length of ordering cycle

$$TVC = \frac{1}{T}\left\{\frac{c_1 A}{\theta(\theta - \lambda)} e^{-\lambda t_1}\left[e^{\theta t_1} - 1 - \frac{\theta}{\lambda}\left(e^{\lambda t_1} - 1\right)\right] + c_2 A\left[\frac{e^{(\theta-\lambda)t_1} - 1}{\theta - \lambda} - \frac{1 - e^{-\lambda t_1}}{\lambda}\right] + c_3 D\left(\frac{c_4}{\lambda} + c_5\right)\left[T - t_1 - \left(\frac{\ln[1 + \delta(T - t_1)]}{\delta}\right)\right]\right\}$$

$$TVC = \frac{1}{T}\left\{\frac{A(c_1 + \theta c_2)}{\theta(\theta - \lambda)}\left[e^{(\theta-\lambda)t_1} - (\theta - \lambda)t_1 - 1\right] - \frac{A(c_1 + \theta c_2)}{\theta\lambda}\left[1 - \lambda t_1 - e^{-\lambda t_1}\right] + c_3 + \frac{D(c_4 + \delta c_5)}{\delta}\left[T - t_1 - \frac{\ln[1 + \delta(T - t_1)]}{\delta}\right]\right\} \ldots\ldots\ldots (12)$$

The objective of the model is to determine the optimal values of t_1 and T in order to minimize the average total cost per unit time, TVC. The optimal solutions t_1^* and T^*need to satisfy the following equations:

$$\frac{\partial TVC}{\partial t_1} = \frac{1}{T}\left\{\frac{A(c_1 + \theta c_2)}{\theta}\left[e^{(\theta-\lambda)t_1} - e^{-\lambda t_1}\right] - \frac{D(c_4 + \delta c_5)}{\delta}\left[1 - \frac{1}{1 + \delta(T - t_1)}\right]\right\} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad ,,,,,,,,,,,,,,,.(13)$$

and

$$\frac{\partial TVC}{\partial T} = \frac{1}{T^2}\left\{\frac{D(c_4 + \delta c_5)}{\delta}\left[\frac{(T - t_1)(\delta t_1 - 1)}{1 + \delta(T - t_1)} + \frac{1}{\delta}\ln[1 + \delta(T - t_1)]\right] - \frac{A(c_1 + \theta c_2)}{\theta(\theta - \lambda)}\left[e^{(\theta-\lambda)t_1}\right] - c_3\right\} = 0 \ldots\ldots\ldots\ldots \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

Particle Swarm Optimization (PSO)

The PSO algorithm is inspired by the natural's warm behavior of birds and fish. It was introduced by Eberhart and Kennedy in 1995 as an alternative to other ECTs, such as Ant Colony Optimization, Genetic Algorithms (GA) or Differential Evolution (DE). Each particle in the population represents a possible solution of the optimization problem, which is defined by its cost function. In each iteration, a new location (combination of cost function parameters) of the particle is calculated based on its previous location and velocity vector(velocity vector contains particle velocity for each dimension of the problem).

The PSO algorithm works by simultaneously maintaining several candidate solutions in the search space. During each iteration of the algorithm, each candidate solution is evaluated by the objective function being optimized, determining the fitness of that solution. Each candidate solution can be thought of as a particle "flying" through the fitness landscape finding the maximum or minimum of the objective function.

Initially, the PSO algorithm chooses candidate solutions randomly within the search space. It should be noted that the PSO algorithm has no knowledge of the underlying objective function, and thus has no way of knowing if any of the candidate solutions are near to or far away from a local or global maximum. The PSO algorithm simply uses the objective function to evaluate its candidate solutions, and operates upon the resultant fitness values.

Each particle maintains its position, composed of the candidate solution and its evaluated fitness, and its velocity. Additionally, it remembers the best fitness value it has achieved thus far during the operation of the algorithm, referred to as the individual best fitness, and the candidate solution that achieved this fitness, referred to as the individual best position or individual best candidate solution. Finally, the PSO algorithm maintains the best fitness value achieved among all particles in the swarm, called the global best fitness, and the candidate solution that achieved this fitness, called the global best position or global best candidate solution.

The PSO algorithm consists of just three steps, which are repeated until some stopping condition is met:

1. Evaluate the fitness of each particle
2. Update individual and global best fitness's and positions

3. Update velocity and position of each particle
4. Repeat the whole process till the

The first two steps are fairly trivial. Fitness evaluation is conducted by supplying the candidate solution to the objective function. Individual and global best fitness's and positions are updated by comparing the newly evaluated finesses against the previous individual and global best fitness's, and replacing the best fitness's and positions as necessary.

The velocity and position update step is responsible for the optimization ability of the PSO algorithm. The velocity of each particle in the swarm is updated using the following equation:

$$v(i+1) = w * v(i) + c_1 * (pBest - x(i)) + c_2 * \left(gBest - x(i)\right) \ldots\ldots\ldots (15)$$

$Where:$
$v(i+1)\ -\ New\ velocity\ of\ a\ particle.$
$v(i)\ -\ Current\ velocity\ of\ a\ particle.$
$c_1\ , c_2\ -\ Priority\ factors.$
$pBest\ -\ Best\ solution\ found\ by\ a\ particle.$
$gBest\ -\ Best\ solution\ found\ in\ a\ population.$
$x(i)\ -\ Current\ position\ of\ a\ particle.$

The new position of a particle is then given by (16), where $x(i\ +\ 1)$ is the new position:

$$x(i+1) = x(i) + v(i+1) \ldots\ldots\ldots\ldots..(15)$$

Each of the three terms $(w * v(i), c_1 * (pBest - x(i))\ and\ c_2 * \left(gBest - x(i)\right)$of the velocity update equation have deferent roles in the PSO algorithm. The first term w is the inertia component, responsible for keeping the particle moving in the same directionit was originally heading. The value of the inertial coefficient w is typically between 0.8 and 1.2, which caneither dampen the particle's inertia or accelerate the particle in its original direction. Generally, lower values of the inertial coefficient speed up the convergence of the swarm to optima, and higher values of the inertial coefficient encourage exploration of the entire search space.

The second term $c_1 * \left(pBest - x(i)\right)$called the cognitive component, acts as the particle's memory, causingit to tend to return to the regions of the search space in which it has experienced high individual fitness. The cognitive coefficient c_1 is usually close to 2, and affects the size of the step the particle takes toward its individual best candidate solution $pBest$.

The third term$c_2 * \left(gBest - x(i)\right)$, called the social component, causes the particle to move to the best region the swarm has found so far. The social coefficient c_2 is typically close to 2, and represents the sizeof the step the particle takes toward the global best candidate solution $gBest$ the swarm has found up until that point.

Simulation and Results

To test the meta-heuristic algorithm for finding the optimal solution of the problem discussed in section (3), the equation (13) and (14) is taken as the fitness function, while the values of t_1 and T are to be searched for satisfying the fitness function. The other variables are taken as follows:

Variable Name	Variable Value
A	12
θ	0.08
δ	2
λ	0.03
c_1	0.5
c_2	1.5
c_3	10
c_4	2.5
c_5	2
D	8

The exact solution found analytically in [1] was $t_1 = 1.4775$and $T = 1.8536$. Which provides the optimal maximum inventory level $W\ =\ 18.401$ units, the optimal order quantity $Q\ =\ 20.1183$ units and the minimum average total cost per unit time $TVC\ =\ \$11.1625$.

While the PSO based technique with following configuration

Variable Name	Variable Value
c_1 (PSO Const.)	2
c_2 (PSO Const.)	1
w_{max}	0.9
w_{min}	0.1
Total Particles	100
Maximum Iterations	100

The equation (12) can be directly selected as objective function for the PSO algorithm hence it avoids the complex differentiations required for analytical methods.

Using the equation (12) as objective function PSO provides the minimum value of $TVC\ =\ \$11.1625$, at $t_1\ =\ 1.4752$ and $T\ =\ 1.8516$, which is exactly same as found by a nalytical solution.

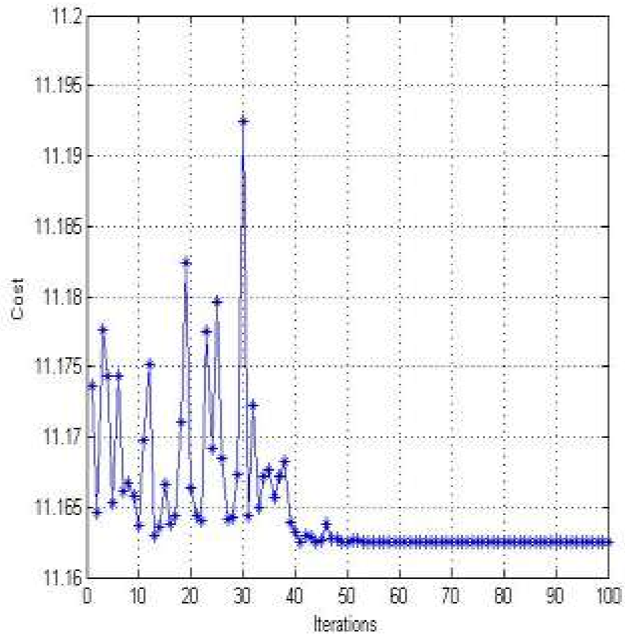

Figure 1: the value of objective function (fitness value) at every iteration of PSO.

Also if we try to use the objective function by combining the equation (13) and (14)

$$objective\ function = abs\left(eq(13)\right) + abs\left(eq(14)\right)$$

Achieves the solution $t_1 = 1.4773$and$T = 1.8534$ which is very close to analytical solution.

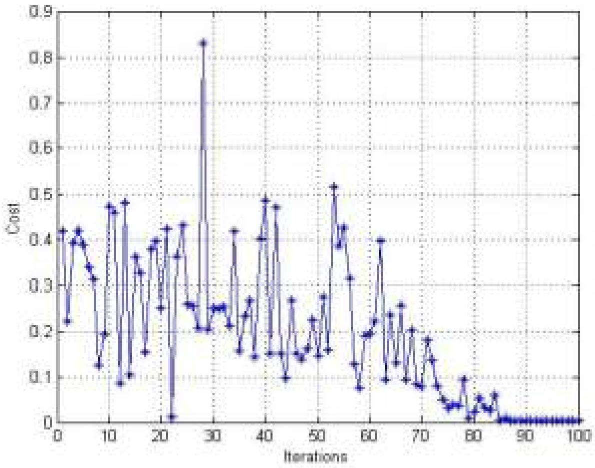

Figure 2: the value of objective function (fitness value) at every iteration of PSO. The bet fitness value achieved in the process is $8.7521 * 10^{-04}$ which almost equal to 0.

Conclusion

This paper presented a meta-heuristic approach for efficient solution of classical economic order quantity (EOQ) model problem. The simulation results shows that the proposed approach can solve the EOQ problem efficiently and can be used as an alternative approach with lower mathematical complexity especially for complex EOQ models where analytical solution required extensive analysis.

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