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E: ISSN NO.: 2349-980X **Shrinkhla Ek Shodhparak Vaicharik Patrika**

An Imperfect Model With Preservation Technology, Variable Holding Cost Under Storage Capacity

Abstract

In this paper, we develop a two-warehouse production–inventory model with imperfect quality items. Shortages are allowed and partially backlogged. The lot sizing problem is then to find the optimal production while minimizing the manufacturer's total cost over the planning horizon. Finally, numerical examples will be used to illustrate the results.

Keyword: E-commerce, Business, Industry, Key Factor, Trends, Future of Commerce, Digital India.

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Introduction

In the last few decades, the development of inventory control models and their uses are popularized by academicians as well as industries.However, one of the weaknesses of current inventory models is the unrealistic assumption that all items produced are of good quality. Defective items, as a result of considering imperfect quality production process were initially considered by Rosenblatt and Lee (1986), Porteus (1986). Besides imperfect quality assumption in production process, other factors such as damages and breakages during the handling process may also result in defective items. The above consideration was included in Salameh and Jaber (2000) who were the few authors who presented a model for items with imperfect quality. Later, Cardenas-Barron (2000) corrected a mistake in the final formulae of Salameh and Jaber's model. Goyal and Cardenas-Barron (2002) then reconsidered the work done in (2000) and presented a practical approach for determining the optimal lot size. They assumed that poor items are withdrawn from stock and no shortage was allowed. Wee (1993) developed an economic production plan for deteriorating items with partial backordering, but he assumed perfect quality.Gupta and Chakraborty (1984) considered the reworking option of rejected items. They considered recycling from the last stage to the first stage and obtained an economic batch quantity model. Cheng (1989) validates Porteus's model by including the learning effects on setup frequency and process quality. Rosenblatt and Lee (1986) assumed that the time from the beginning of the production run until the process goes out of control is exponential and that defective items can be reworked instantaneously at a cost and kept in stock. Jamal et al. (2004) assumed that all defective products could be reworked. Recently, Ben-Daya et al. (2006) developed integrated inventory inspection models with and without replacement of nonconforming items discovered during inspection. Inspection policies include no inspection, sampling inspection, and 100% inspection. They proposed a solution procedure for determining the operating policies for inventory and inspection consisting of order quantity, sample size, and acceptance number. Salameh and Jaber (2000) surveyed an EOQ model where each lot contains a certain percentage of defective items with a continuous random variable. They also considered that imperfect items could be sold as a single batch at a reduced price by the end of 100% inspection but they did not address the impact of the reject and the rework and ignored the factor of when to sell.

In a subsequent paper, Lee (1987) considered a joint lot sizing and inspection policy is studied under an economic order quantity model where a random proportion of units are defective. Salameh and Jaber (2000) hypothesized a production/inventory situation where items, received or produced, are not of perfect quality. Items of imperfect quality, not necessarily defective, could be used in another production/inventory situation, that is, less restrictive process and acceptance control. They extended the traditional EPQ/EOQ model by accounting for imperfect quality items when using the EPQ/EOQ formulae. Chan et al. (2003) provided a framework to integrate lower pricing, rework and reject situations into a single EPQ model. A 100% inspection is performed in order to identify the amount of good quality items, imperfect quality items and defective items in each lot. Papachristos and Konstantaras (2006) looked at the issue of non-shortages in models with proportional imperfect quality, when the proportion of the imperfects is a random variable and revised the papers of Salameh and Jaber (2000) and Chan et al. (2003). Hayek and Salameh (2001) presented an inventory model of shortage and backlog that considers rework of defective products. Zhang and Gerchak (1980) considered a joint lot sizing and inspection policy studied under an EOQ model where a random proportion of units are defective. Wee et al. (2007) developed an optimal inventory model for items with imperfect quality and shortage backordering. They assumed that all customers are willing to wait for new supply when there is a shortage. Ouyang et al. (2002) investigated the lot size, reorder point inventory model involving variable lead time with partial backorders, where the production process is imperfect. Francis Leung (2007) proposed an EPQ model with a flexible and imperfect production process. He formulated this inventory decision problem using geometric programming. Friemer et al. (2006) investigated the effect of imperfect yield on economic production quantity decisions. Ouyang and Chang (2000) investigated the impact of quality improvement on the modified lot size reorder point models involving variable lead time and partial backorders. Chiu (2003) considered the effects of the reworking of defective items on the economic production quantity (EPQ) model with allowed backlogging. Urban (1982) proposed a finite replenishment inventory model in which the demand of an item is a deterministic function of price and advertising expenditures. The formulated models also incorporate learning effects and the possibility of defective items in the production process. Ben Daya (1989) proposed multi-stage lot sizing models for imperfect production processes. Lee (2005) developed a cost/benefit model for supporting investment strategies about inventory and preventive maintenance in an imperfect production system. The effect of such investments on the return is expressed as a function of measurable variables. Using this model, the decision maker can decide whether investments in inventory and preventive maintenance are necessary and how much to invest.

Maximum physical goods undergo decay or deterioration over time. Fruits, vegetables and food items suffer from depletion by direct spoilage while stored. Highly volatile liquids such as gasoline, alcohol and turpentine undergo physical depletion over time through the process of evaporation. Electronic goods, radioactive substances, photographic film, grain, etc. deteriorate through a gradual loss of potential or utility with the passage of time. So decay or deterioration of physical goods in stock is a very realistic feature and inventory researchers felt the necessity to use this factor into consideration. Ghare and Schrader (1963) developed a model for an exponentially decaying inventory. An order level inventory model for items deteriorating at a constant rate was presented by Shah and Jaiswal (1977), Aggarwal (1978), Dave and Patel (1981). Inventory models with a time dependent rate of deterioration were considered by Covert and Philip (1973, 1974), Mishra (1975) and Deb and Chaudhuri (1986). Some of the recent work in this field has been done by Chung and Ting (1993), Fujiwara (1993), Hariga (1996), Hariga and Benkherouf (1994), Jalan, et al. (1996), Chakraborty and Chaudhuri (1997), Giri and Chaudhuri (1997), and Jalan and Chaudhuri (1999).

A notable extension of stock-dependent demand models incorporates variable holding costs. Weiss (1982) noted that variable holding costs are appropriate when the value of an item decreases the longer it is in stock; Ferguson et al. (2007) recently indicated that this type of model is suitable for perishable items in which price markdowns or removal of aging product are necessary. Alfares (2007) states that more sophisticated storage facilities and services may be needed for perishable items if they are kept for longer periods of time. Goh (1994) first considered a stock-dependent demand model with variable holding costs, and assumed that the unit holding cost is a nonlinear continuous function of the time the item is in stock or a nonlinear continuous function of the inventory level. Giri and Chaudhuri (1998) extended this model to account for perishable products. Chang (2004) then amended the Giri and Chaudhuri model to utilize a profit-maximization objective and to allow for a positive inventory level at the end of the order cycle. Alfares (2007) investigated the situation in which the variable holding costs are discrete in nature—a step function of the time in stock—with successively increasing costs.

Few years ago, investing on preservation technology (PT) for reducing deterioration rate has little attention. The reflection of PT is important due to speedy changes and the fact that PT can reduce the deterioration rate significantly. Hsu, Wee and Teng (2010), Singh, Jain and Dem (2013) developed a model with preservation technology. Joaquin Sicilia et al. (2014) derived an inventory model for deteriorating items with shortages and time-varying demand.

1. Assumptions and Notations

1.1 Assumptions

1. OW has the limited capacity while RW has unlimited capacity.
2. Imperfect production is taken into this consideration.
3. Demand rate is constant.
4. Shortages are allowed and partially backlogged.
5. Preservation technology for decaying items is taken into consideration.
6. Inspection costs are considered in both OW and RW separately.
7. Holding cost is time dependent in both OW and RW.
8. Holding cost in RW is higher than that of OW.

2.2 Notations

P is the production rate.

d is the demand rate.

k is the original deterioration rate.

B is the backlogging parameter.

ξ is the preservation technology (PT) cost for reducing deterioration rate to preserve the products, $\xi \geq 0$.

m (ξ) is the reduced deterioration rate, a function of ξ defined by $m(\xi)=k(1-e^{-a\xi})$, a ≥ 0, where a is the simulation coefficient representing the percentage increase in m(ξ).
Cp is the production cost per unit per unit time.
h1 is the holding cost for own warehouse (OW) per unit per unit time.
h2 is the holding cost for rented warehouse (RW) per unit per unit time.
C0 is the inspection cost for OW per unit per unit time.
CR is the inspection cost for RW per unit per unit time.
d1 is the damaged item cost per unit per unit time.
s is the shortage cost per unit per unit time.
l is the lost sale cost per unit per unit time.
W is the own warehouse capacity.
A is the rework cost.

3. Model Formulation

In this model, during the time interval [0,t1], production starts and stock is kept in OW and inventory increases due to the combined effect of demand, production and deterioration. At t = 0, perfect and imperfect units are stored in OW until its capacity W is full. Remaining perfect and imperfect items are stored in rented warehouse at t = t1, and production stops at t = t1.Now, during the time interval [t2, t3] and [t3, t4], inventory decreases due to effect of demand and deterioration. While at time [t1,t3] items are depletes due to deterioration occurs.

As per above description, the inventory level governed the following equations:

$$I_1'(t)=[m(\xi)-k]I_1(t)+P-d,\quad 0\le t\le t_1 \qquad (1)$$

$$I_2'(t)=[m(\xi)-k]I_2(t)+P-d,\quad t_1\le t\le t_2 \qquad (2)$$

$$I_3'(t)=[m(\xi)-k]I_3(t)-d,\quad t_2\le t\le t_3 \qquad (3)$$

$$I_4'(t)=[m(\xi)-k]I_4(t),\quad t_1\le t\le t_4 \qquad (4)$$

$$I_5'(t)=[m(\xi)-k]I_5(t)-d,\quad t_3\le t\le t_4 \qquad (5)$$

$$I_6'(t)=-Bd,\quad t_4\le t\le T \qquad (6)$$

With initial conditions

$$I_1(0)=0, I_2(t_1)=0, I_3(t_3)=0, I_4(t_1)=W, I_5(t_4)=0, I_6(t_4)=0 \qquad (7)$$

Solutions of these equations are:

$$I_1(t)=\frac{[P-d]}{[k-m(\xi)]}[1-e^{-[k-m(\xi)]t}] \qquad (8)$$

$$I_2(t)=\frac{[P-d]}{[k-m(\xi)]}[1-e^{[k-m(\xi)](t_1-t)}] \qquad (9)$$

$$I_3(t)=\frac{d}{[k-m(\xi)]}[e^{[k-m(\xi)](t_3-t)}-1] \qquad (10)$$

$$I_4(t)=We^{[k-m(\xi)](t_1-t)} \qquad (11)$$

$$I_5(t)=\frac{d}{[k-m(\xi)]}[e^{[k-m(\xi)](t_4-t)}-1] \qquad (12)$$

$$I_6(t)=Bd(t_4-t) \qquad (13)$$

The present worth of the production Cost (PD) is given by

$$PD=C_p\int_0^{t_2}Pdt \quad =PC_pt_2 \qquad (14)$$

The present worth of the holding cost for OW is given by

$$HC_1 = (h_1 + \alpha\, t)[\int_0^{t_1} I_1(t)dt + \int_{t_1}^{t_3} I_4(t)dt + \int_{t_3}^{t_4} I_5(t)dt]$$

$$= h_1 \begin{bmatrix} (P-d)\frac{t_1^2}{2} + W\left\{\left(t_1t_3 - \frac{t_3^2}{2} - \frac{t_1^2}{2}\right) + [k - m(\xi)]\left(t_1t_3 - \frac{t_3^2}{2} - \frac{t_1^2}{2}\right)\right\} \\ d\left(\frac{t_3^2}{2} + \frac{t_4^2}{2} - t_4t_3\right) \end{bmatrix}$$
$$+\alpha \begin{bmatrix} (P-d)\frac{t_1^3}{3} + W\left\{\left(\frac{t_1t_3^2}{2} - \frac{t_3^3}{3} - \frac{t_1^3}{6}\right) + [k - m(\xi)]\left(\frac{t_1t_3^2}{2} - \frac{t_3^3}{3} - \frac{t_1^3}{6}\right)\right\} \\ d\left(\frac{t_3^3}{3} + \frac{t_4^3}{6} - \frac{t_4t_3^2}{2}\right) \end{bmatrix} \quad (15)$$

The present worth of the holding cost for RW is given by

$$HC_2 = (h_2 + \alpha\, t)[\int_{t_1}^{t_2} I_2(t)dt + \int_{t_2}^{t_3} I_3(t)dt]$$

$$= h_2\left[(P-d)\left(\frac{t_2^2}{2} + \frac{t_1^2}{2} - t_1t_2\right) + d\left(\frac{t_2^2}{2} + \frac{t_3^2}{2} - t_3t_2\right)\right]$$
$$+\alpha\left[(P-d)\left(\frac{t_2^3}{3} + \frac{t_1^3}{6} - \frac{t_1t_2^2}{2}\right) + d\left(\frac{t_2^3}{3} + \frac{t_3^3}{6} - \frac{t_3t_2^2}{2}\right)\right] \quad (16)$$

The present worth of the damaged cost is given by

$$D_0 = d_1[\int_0^{t_2} P(1-\delta)dt - \int_{t_2}^{t_4} d.dt]$$
$$= d_1\left[P(1-\delta)t_2 - d(t_4 - t_2)\right] \quad (17)$$

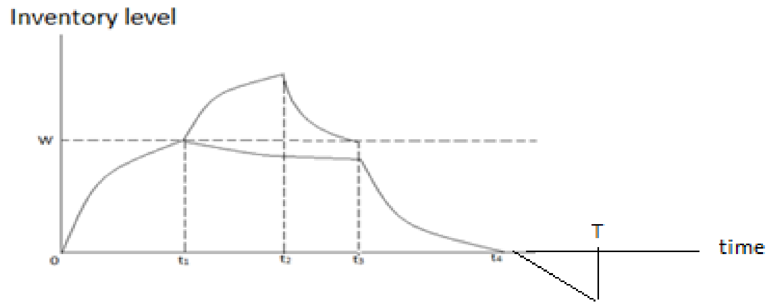

Fig 1: Graphical representation of two warehouse production model

The present worth of the inspection cost for OW is given by

$$IC_0 = C_0\int_0^{t_1} Pdt = C_0Pt_1 \quad (18)$$

The present worth of the inspection cost for OW is given by

$$IC_R = C_R \int_{t_1}^{t_2} P dt = C_R P(t_2 - t_1) \quad (19)$$

The present worth of the rework cost is given by

$$RC = A\left[\int_0^{t_1} P dt + \int_{t_1}^{t_2} P dt\right] = APt_2 \quad (20)$$

The present worth of the shortage cost is given by

$$SC = s\left[\int_{t_1}^{t_2} I_6(t) dt\right] = sBd\left[t_4 t_2 - \frac{t_2^2}{2} - \frac{t_1^2}{2} - t_4 t_1\right] \quad (21)$$

The present worth of the lost sale cost is given by

$$LC = l\left[\int_{t_1}^{t_2} [1-B] d.dt\right] = l(1-B)d(t_2 - t_1) \quad (22)$$

The present worth of the total cost is given by

$$TC = [C_p.PD + h_1.HC_1 + h_2.HC_2 + S.D_0 + C_0.IC_0 + C_R.IC_R + A.RC + \xi + s.SC + v.LC] \quad (23)$$

4. Solution Procedure

The total annual cost has the two variables t4 and ξ. To minimize the total annual cost, the optimal values of t4 and ξ can be obtained by solving the following equations simultaneously

$$\frac{\partial TC}{\partial t_4} = 0 \quad (24)$$

$$\frac{\partial TC}{\partial \xi} = 0 \quad (25)$$

Provided, they satisfy the following conditions and

$$\left.\begin{aligned} &\frac{\partial^2 TC(t_4,\xi)}{\partial t_4^2} > 0, \frac{\partial^2 TC(t_4,\xi)}{\partial \xi^2} > 0 \\ &\left(\frac{\partial^2 TC(t_4,\xi)}{\partial t_4^2}\right)\left(\frac{\partial^2 TC(t_4,\xi)}{\partial \xi^2}\right) - \left(\frac{\partial^2 TC(t_4,\xi)}{\partial t_4 \partial \xi}\right)^2 > 0 \end{aligned}\right\} \quad (26)$$

To minimize the objective function, the optimal solution of t4 and ξ can be obtained from the equation (24) and (25). All these equations are solved numerically with the help of Software.

5. Numerical Examples

To exemplify the above model numerically, we have considered the following data given in Table 1 in appropriate units

$$P = 250, d = 150, C_p = 8, W = 50, C_0 = 1, C_R = 2, l = 0.6, A = 2, B = 0.04$$

Output results are

$$t_1^* = 0.235767, t_2^* = 0.337889, t_3^* = 0.57875, t_4^* = 1.06372, \xi^* = 65.7865, TC^* = 5457.66$$

6. Sensitivity Analysis

Table 1: Effect of Parameter P on proposed policy

P	t1*	t2*	t3*	t4*	ξ*	TC*
260	0.225689	0.324348	0.588990	1.05873	66.8934	5349
270	0.215867	0.313234	0.571323	1.04545	67.2434	5289
280	0.209897	0.300909	0.561213	1.03547	68.3423	5123

290	0.192245	0.290933	0.541323	1.02098	69.8799	5045
300	0.183449	0.280983	0.539321	1.01996	70.8977	4999

Table 2: Effect of Parameter d on proposed policy

D	t1*	t2*	t3*	t4*	ξ*	TC*
160	0.239888	0.341556	0.583667	1.05446	66.0332	5556
170	0.240342	0.351627	0.594345	1.03802	67.2121	5591
180	0.256908	0.362604	0.605932	1.01686	68.3321	5623
190	0.265663	0.374054	0.616371	1.00897	69.8989	5678
200	0.270212	0.386599	0.627329	0.90065	70.0988	5735

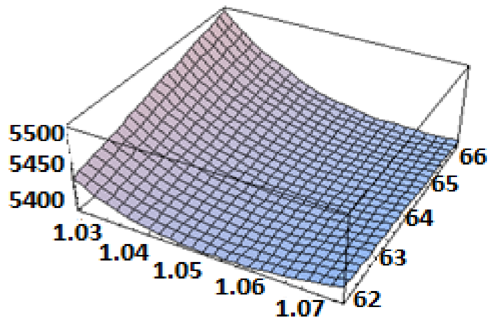

Fig 2: Convexity of the total cost w.r.t. t4 and ξ

Table 3: Effect of Parameter C0 on proposed policy

C0	t1*	t2*	t3*	t4*	ξ*	TC*
1.2	0.235767	0.337521	0.562233	1.05900	64.3231	5480.
1.4	0.235767	0.336127	0.561989	1.05634	62.0987	5498.
1.6	0.235767	0.328981	0.551434	1.04821	60.8776	5512.
1.8	0.235767	0.328675	0.550783	1.03782	58.4121	5667.
2.0	0.235767	0.317990	0.549731	1.03243	56.1232	5721.

Table 4: Effect of Parameter CP on proposed policy

Cp	t1*	t2*	t3*	t4*	ξ*	TC
9	0.235767	0.329232	0.561567	1.06232	64.5656	5460.6
10	0.235767	0.328765	0.560884	1.05999	63.6786	5465.8
11	0.235767	0.318977	0.559861	1.05766	62.3435	5487.5
12	0.235767	0.317456	0.558977	1.04533	61.8966	5524.4
13	0.235767	0.306778	0.558715	1.03221	60.4756	5589.3

7. Observations

1. As we increase the rate of production, t1*, t2*, t3*, t4*, total cost decreases while preservation cost increases.
2. As we increase the rate of demand, t1*, t2*, t3*, preservation cost and total cost increases whereas t4* decreases.
3. As we increase the inspection cost for OW, t1* remains same, t2*, t3*, t4* and preservation cost decreases while total cost increases.
4. As we increase the inspection cost for RW, t1* remains same, t2*, t3* decreases and t4*, preservation cost, total cost increases.

Conclusion

We have considered an imperfect production with variable holding cost under two storage capacity. Shortages are allowed and partially backlogged. While this research represents an important generalization of existing inventory models, further investigation can be conducted in a number of areas. Potential topics for future research include extending other models with continuously variable holding costs to the discretely variable case. Incorporating a stochastic demand rate would be notable, since the actual level of holding cost attained would be uncertain. Finally, empirical research should be conducted to determine which products reflect variable holding costs and to see precisely how (discretely or continuously) the holding costs vary over time.

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