

# Abnormal behavior of 1D plasma photonic crystal at Low Plasma Frequency

## Abstract

In this paper, the electromagnetic properties of one dimensional dielectric-plasma photonic crystals (PPCs) namely, band structure, reflection properties, group velocity and effective group index have been presented at low plasma frequency than the frequencies of incident radiation. Here, two structures PPC1 and PPC2 are considered by choosing SiO<sub>2</sub> and TiO<sub>2</sub> as the materials of dielectric layers of PPC1 and PPC2 respectively. For certain normalized frequency range, the group velocity becomes negative. Because of this abnormal behavior ( $V_g < 0$ ), superluminal flow of photon occurs in the PPCs. Such structures may be considered as a flip-flop as there is positive and negative symmetry of effective group velocity. Such a flip flop may be used to making logic gates, optical switches in optical computing.

**Keywords:** Please add some keywords

## Introduction

Plasma is the wide variety of macroscopically neutral substances which contains many interacting free electrons and ionized atoms or molecules. The collective behavior of plasma is due to long-range columbic forces. After the works of Joannopoulos et.al. and Scalora et.al. photonic crystals, have been getting attention in the field of solid state and optical physics due to their many unique features. The technological applications of photonic crystals are expanding widely due to their controlling properties. Photonic crystals, also known as Photonic Band Gap materials, are materials which have alternate forbidden and allowed band gaps. Plasma photonic crystals consists alternate layers of plasma with certain plasma frequency and dielectric materials. Plasma is the important candidate to design PCs because its dielectric constant is highly dependent on its frequency and can be controlled by its frequency significantly.

## Aim of the Study

In the present communication, the optical properties of a one-dimensional plasma photonic crystal (PPC) containing alternate dielectric and plasma layers has been studied at low plasma frequencies. The dispersion properties, reflection spectra and group velocity of such photonic crystals have been presented using Transfer Matrix Method. It is found that the bandwidth of photonic band gap(s) can be increased or decreased as the function of thickness of plasma layers and group velocity reaches negative values at a certain frequencies of incident wave. Plasma is the useful agent in photonic crystal because it controls the band gaps with its frequency.

## Review of Literature

For the first time, Keskinen et.al. and Hojo et.al. have presented the research on photonic band gaps in plasma photonic crystals using dusty plasma and discharged micro-plasma respectively. This photonic crystal is named as *Plasma Photonic Crystal* (PPC) because of plasma layers. Plasma photonic crystal is a periodic array composed of alternating layers of plasma and dielectric materials in 1-, 2- or 3-dimensions. Hojo and Mase studied plasma photonic crystal and proved that photonic band gap(s) increase or decrease as the function of the width of plasma layers as well as the plasma density. It is also confirmed by Ojha et al. in their communications in Optik (2008) and Progress in Electromagnetics Research (2009).

The theoretical aspect of the band gap structure and absorption-less transmission of electromagnetic wave in one dimensional Plasma Photonic Crystals (PPCs) with alternate layers of micro-plasma and dielectric materials has been presented by Hojo et.al. The dispersion

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## Remarking An Analisation

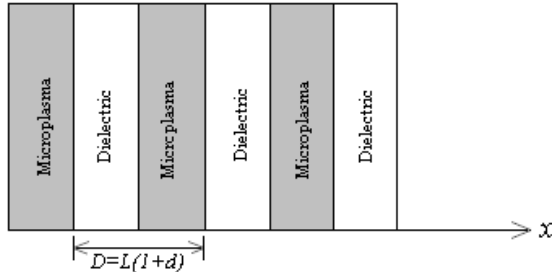
relation can be obtained by solving the Maxwell's equations using a method analogous to Kronig-Penny's model in quantum mechanics. The frequency gap and cutoff appeared in the dispersion relation (plot of wave vector as the function of frequency of incident radiation) of the PPCs. The band gap is found to become wider with the increase of plasma density as well as the width of plasma layers. Marklund et.al. have discovered that the absorption-less transmission can be possible for single layer transmission as well as for two layers at critical plasma frequency and this is considered as Feby-Perot resonance, which is well known phenomenon in optics.

In the present communication, the electromagnetic properties of one dimensional dielectric-plasma photonic crystals (PPCs) namely, band structure, reflection properties; group velocity and effective group index have been reported. For the sake of numerical calculations, two structures PPC1 and PPC2 are considered by choosing SiO<sub>2</sub> and TiO<sub>2</sub> as the materials of dielectric layers of PPC1 and PPC2 respectively. SiO<sub>2</sub> and TiO<sub>2</sub> are two commonly used dielectric materials in semiconductor industry. In the next section numerical formulas for optical properties have been discussed.

### Theoretical Analysis

The structure having alternate layers of dielectric and micro-plasma as shown in Figure 1 has been considered for the present study.

**Figure 1: Periodic variation of plasma and dielectric layers showing 1-D plasma photonic crystals**



The photonic parameters such that dispersion relation, reflectivity, group velocity and effective group index of one-dimensional plasma photonic crystals (PPCs) are computed using by transfer matrix method developed by P. Yeh. The Maxwell wave equations for electromagnetic wave propagating along the x-axis in one-dimensional PPCs may be written as

$$\frac{d^2 E(x)}{dx^2} + k_0^2 \varepsilon(x) E(x) = 0, \quad (1)$$

$$\text{with } \varepsilon(x) = \begin{cases} 1 - \frac{\omega_p^2}{\omega^2}, & -Ld < x < 0, \\ \varepsilon_m, & 0 < x < L, \end{cases} \quad (2)$$

$$\text{and } \varepsilon(x) = \varepsilon(x + D), \quad (3)$$

Where,  $k_0 = \omega/c$  is the wave frequency,  $c$  is the speed of light,  $\omega_p = (e^2 n_p / \varepsilon_0 m)^{1/2}$  is the electron

plasma frequency with plasma density  $n_p$ ,  $\varepsilon_m$  is the dielectric constant of the dielectric material.

The schematic diagram of the spatial variation of bilayers (micro-plasma and dielectric material) is shown in Figure 1, where  $D=L(1+d)$  is the lattice period having the widths of dielectric and micro-plasma being  $L$  and  $Ld$  respectively.

The general solution of the equation (1) for low plasma frequency is given by, i.e. for  $\omega > \omega_p$

$$E(x) = \begin{cases} a_m e^{ik_d x} + b_m e^{-ik_d x}, & -Ld < x < 0, \\ c_m e^{ik_p x} + d_m e^{-ik_p x}, & 0 < x < L, \end{cases} \quad (5)$$

where,  $k_d = \omega/c(\varepsilon_m)^{1/2}$ ,  $\kappa = \omega/c(\omega_p^2/\omega^2 - 1)^{1/2}$  and  $k_p = \omega/c(1 - \omega_p^2/\omega^2)^{1/2}$ , and  $a_m, b_m, c_m$  and  $d_m$  are constants.

By applying the continuity of  $E(x)$  and  $\dot{E}(x)$  at the interfaces of different materials of the structures, the following matrix equation can be obtained

$$\begin{bmatrix} a_{m-1} \\ b_{m-1} \end{bmatrix} = M \begin{bmatrix} a_m \\ b_m \end{bmatrix} \quad (6)$$

Here, the translation matrix is given by

$$M = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \quad (7)$$

Now, according to Bloch's theorem, because of the periodicity in the structure of the PPCs, the electric field vector can be expressed in the form  $E(x) = E_K(x)e^{-ikx}$ , where  $E_K(x)$  is periodic in the lattice period of  $D$ . The constant  $K$  is known as wave number and can be written as

$$K(\omega) = \frac{1}{D} \cos^{-1} \left[ \frac{1}{2} (M_{11} + M_{22}) \right] \quad (8)$$

However, the group velocity [ $V_g(\omega)$ ] of the incident wave for the proposed structure can be expressed by using the following formula given by Sakoda et.al.

$$V_g = \left( \frac{dK}{d\omega} \right)^{-1} \quad (9)$$

And the effective group index of refraction  $n_{eff}(g)$  can be calculated using the group velocity and can be represented by the following formula

$$n_{eff}(g) = \frac{c}{V_g} \quad (10)$$

The expressions for the reflectivity of the PPC structure is given by

$$|R_N|^2 = \frac{|M_{21}|^2}{|M_{21}|^2 + (|\sin(KD) / \sin(NKD)|)^2} \quad (11)$$

## Remarking An Analisation

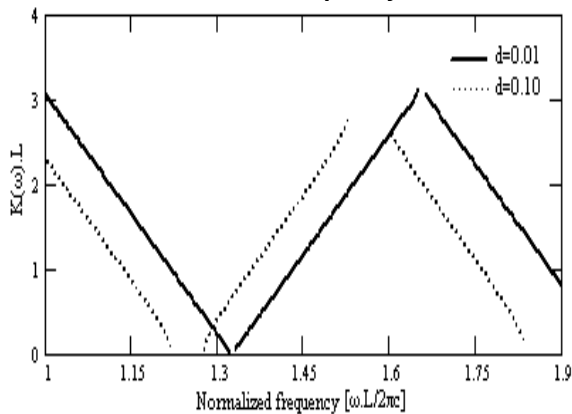
### Results and Discussion

In this section, the band structure, group velocity,  $V_g$ , effective group index,  $n_{eff}(g)$  and reflectivity are computed as the function of normalized frequency ( $\omega L/2\pi c$ ) of one-dimensional plasma photonic crystals (PPCs) for which the frequency of incident radiation,  $\omega > \omega_p$  where  $\omega_p$  is the plasma frequency. There are two PPC structures considered with alternate layers of dielectric and micro-plasma in the present study as -

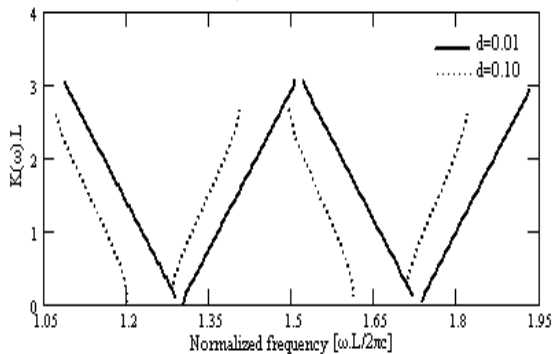
- (i) PPC1: SiO<sub>2</sub>-Plasma
- (ii) PPC2: TiO<sub>2</sub>-Plasma

For both the PPCs, the thickness and frequency of plasma layers is taken as  $Ld$  and  $5.6 \times 10^{11}$  Hz respectively. The thickness and refractive index of dielectric layer are taken as  $L$  and 1.5 (for SiO<sub>2</sub> in PPC1) or 2.3 (for TiO<sub>2</sub> in PPC2) respectively. There are considered two cases of different thickness ratios ( $Ld/L=d$ ) for both the structures. The ratios are 0.01 and 0.10 for the PPC1 and PPC2 respectively. Equations (8) to (11) are used to compute the various optical properties namely band structure, reflectivity, group velocity and effective group index. Figures 2 and 3, shows the variation of normalized wave vector vs normalized frequency called the dispersion relations for PPC1 and PPC2 respectively.

**Figure 2: Dispersion Relation for PPC1: Variation of Normalized Wave Vector  $[K(\omega).L]$  As The Function Of Normalized Frequency**



**Figure 3: Dispersion relation for PPC2: Variation of Normalized Wave Vector  $[K(\omega).L]$  Versus Normalized Frequency**



The corresponding band gaps in the form of numerical values for both PPCs are tabulated in Table 1. From Table 1 it is clear that the forbidden band gaps also increase with the value of  $d$ . For PPC1 the

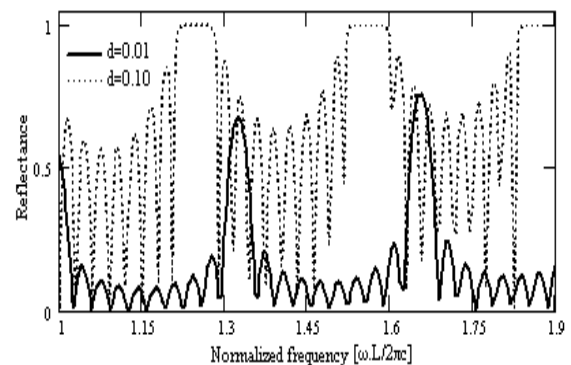
photonic band gap width for  $d=0.10$  is 6 times larger than that of  $d=0.01$  and for PPC2 the photonic band gap width of  $d=0.10$  is 5 times broader than that of  $d=0.01$  respectively.

**Table 1: Numerical values of Photonic band gaps of both PPCs for  $\omega > \omega_p$ .**

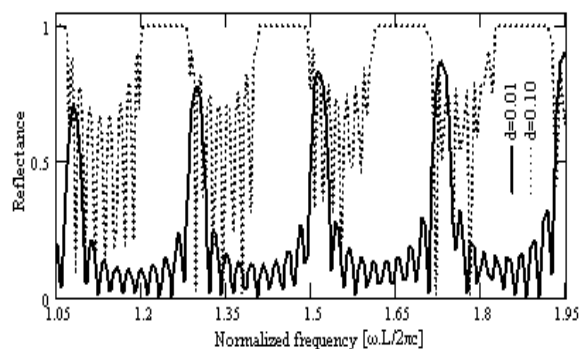
Value of $d$	PPC1		PPC2	
	Band Gap ( $\omega L/2\pi c$ )	Band width	Band Gap ( $\omega L/2\pi c$ )	Band width
0.01	1.320-	0.010	1.290-	0.015
	1.330	0.010	1.305	0.017
	1.650-		1.507-	0.020
	1.660		1.524	
0.10			1.715-	
			1.735	
	1.220-	0.060	1.205-	0.075
	1.280	0.060	1.280	0.084
	1.540-		1.408-	0.094
		1.492		
		1.625-		
		1.719		

Here, it is also noted that with the value of  $d$ , the band gap also shifts towards the lower normalized frequency region. Also, the mid points of band gaps are not lie in same region. Figures 4 and 5 represent the reflectivity curves for PPC1 and PPC2 respectively. The 100% reflection ranges corresponding to the band gaps shifts towards the lower normalized frequency region with the increment in  $d$ .

**Figure 4: Reflection Spectra: Variation of Reflectance versus Normalized Frequency for PPC1.**



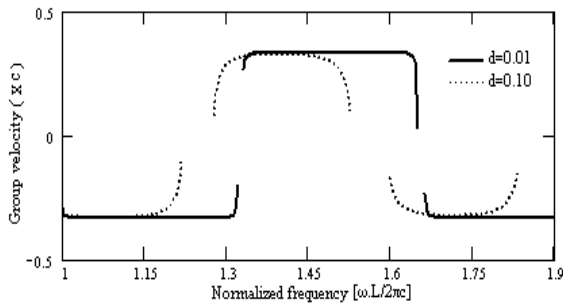
**Figure 5: Reflection Spectra: Variation of Reflectance Versus Normalized Frequency for PPC2.**



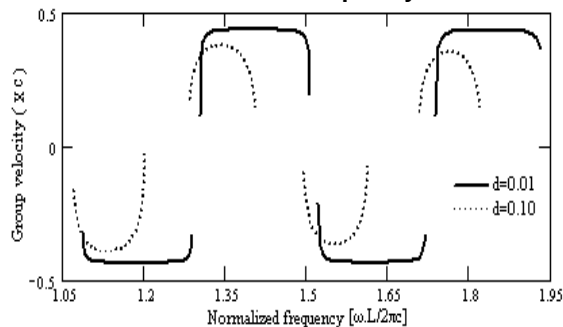
## Remarking An Analisation

The group velocities and effective group index of both the structures are shown in Figures 6, 7 and Figures 8 and 9 respectively. The group velocities for PPC1 are positive and negative for the certain ranges of normalized frequencies as found in Figures 6 and 7. There is no perfect symmetries in these positive and negative values for  $d=0.01$ . On the other hand if the thickness  $d$  increased, the structures attain best symmetrical results. The effective group index is depicted as the function of normalized frequency in the Figures 8 and 9 respectively for both PPCs. It is found that the values of effective group index for  $d=0.01$  have high positive than that of  $d=0.10$  in case of both PPCs. But the magnitude of negative group velocities for  $d=0.10$  have high values than that of the structure with  $d=0.01$ . In this way the appropriate value of  $d$  can be selected for achieving high magnitude of positive as well as negative values of group velocity. Thus, PPCs may be used as a flip flop in optical communication because of their up-down symmetries. Also, the band-gaps can be tuned by controlling the value of  $d$ .

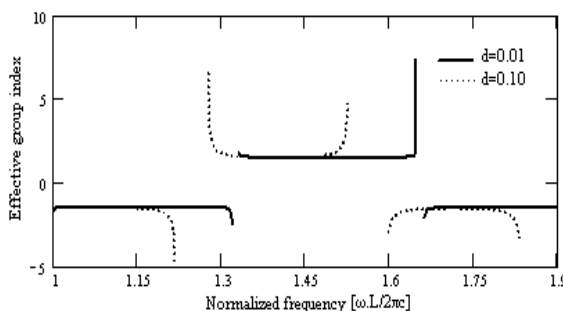
**Figure 6: Variation of group velocity as the function of normalized frequency for PPC1.**



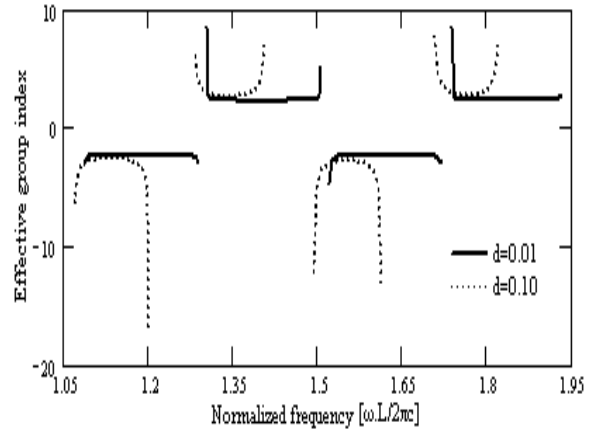
**Figure 7: Variation of group velocity as the function of normalized frequency for PPC2.**



**Figure 8: Variation of effective group index of refraction as the function of normalized frequency for PPC1.**



**Figure 9: Variation of effective group index of refraction as the function of normalized frequency for PPC2**



### Conclusion

This study demonstrates that the thickness of plasma layers in PPCs play an important role for control the forbidden band gaps. So investigators can use a PPC as a broadband reflector and frequency selector/rejecter by choosing appropriate values of plasma thickness and suitable material for dielectric layers. For PPC1, the mid-point of a band gap shifts towards the higher side of the normalized frequency, whereas for PPC2 the mid-point of a band gap shifts towards the opposite side i.e. lower normalized frequency range as increasing the thickness of the plasma layers. The group velocity becomes negative for certain normalized frequency range. The photons may attain superluminal flow in the PPCs due to this abnormal behavior ( $V_g < 0$ ). Because of this positive and negative symmetry of effective group velocity such structures may be used as a flip-flop in photonic computing.

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