P: ISSN NO.: 2394-0344 RNI No.UPBIL/2016/67980 VOL-3* ISSUE-5* (Part-1) August- 2018
E: ISSN NO.: 2455-0817 Remarking An Analisation

# Is Chaos Present in the Stock Market


## Abstract

Academic and applied researchers studying stock markets and other economic series have become interested in the topic of chaotic dynamics. The possibility of chaos in stock market questions the use of linear models for predicting stock market movements. This paper examines the presence of chaos in the stock market by applying Lyapunov exponent on CAPM (Capital Asset Pricing Model). The positive value of Lyapunov exponent of returns indicates that the stock market is chaotic. The non-linear nature of returns recommends the use of non linear methods for predicting stock market movements rather than linear models in practice. The paper strengthens the idea that non-linear models would provide the answers to stock price movements better than the linear models.


**Keywords:**

**Kamini Bhutani**
Associate Professor,
Deptt.of Commerce,
Bharati College,
University of Delhi
Janak Puri, New Delhi

**Introduction**

Every time people have been trying to make their life structured and organized with the help of legal system, bureaucracy and organization charts. Yet, the world is not orderly, nature is not orderly nor the human created institutions like capital market. But we do not understand how they work. Financial economists have been dominated by a linear paradigm that is for every action there is a proportionate reaction. The markets are rarely so orderly; there is an exponential over reaction to action. This type of characteristics leads a system far from equilibrium which does not fit the Efficient Market Hypothesis (EMH) which has dominated quantitative investment finance.

**Objective of the Study**

Predicting the future is always a fascinating adventure and such attempts has been made more in respect of stock prices than any other branch of finance. The predictions of stock price movements have been made on traditional statistical forecasting methods for many decades. Linear models have been the basis of such traditional statistical forecasting. In the absence of exact knowledge of laws governing fluctuations in financial markets, the accuracy of such prediction depends on the discovery of empirical regularities in the financial time series.

What has been realized after many attempts is that several variables that determine stock prices are not independent, but are quite often dependent and the time when a particular variable starts moving in one direction or the other is uncertain. Moreover there seems to be no regularity in changes in price due to change in other variables and price movements in the market resembles chaos. The problem, therefore, is to find the chaotic nature of the stock market.

To begin with, the relation of current prices to future prices will not be linear but non-linear. This non-linearity implies that past price changes can have wide ranging effects on future prices. Probably, the apparent complexity in the stock market may be due to non-linear interaction between several significant variables. Therefore, it would be appropriate to understand the nature of the stock market using chaos theory.

The motivation for undertaking this study is not only the dearth of research in this domain but also the potential implications of such a study for players in this market. The objective of this paper is to measure the sensitive dependence upon initial conditions, which is a characteristic of chaotic behavior. The sensitive dependence on initial condition can be explained using Lyapunov exponent.

EMH utilizes statistical methods developed by Louis Bachelier back in 1900. This hypothesis assumes that stocks are priced so that all public information, both fundamental and price history, is already discounted. Prices, therefore, move only when new information is received. An efficient market cannot be gamed because not only the prices reflect known information, but the large number of investors will ensure that the prices are fair. Thus EMH assumes that investors are rational, orderly and tidy. They are risk averse and require mean/variance (risk return) efficiency. They know, in a collective sense, what information is important and what is not. Then after digesting the information and assessing the risk involved, the collective consciousness of the markets finds an equilibrium price. This hypothesis holds that it is impossible to make a profit through trading strategies in the long run. Prices only change due to unexpected news but are immediately arbitraged away by investors until a stable state of equilibrium is reached.

After EMH, Modern Portfolio Theory (MPT) was developed by Markowitz (1952). He made the distribution of possible returns, as measured by its variance, the measure of riskiness of the portfolio. Formally, the variance is defined by the following formula:

$$\sigma^2 = \sum_{i=1}^{\infty}(r_i - r_u)2$$

$$\begin{aligned} Where\ \ \sigma^2 &= \text{var}iance \\ r_u &= mean\ return \\ r_i &= return\ observation \end{aligned}$$

Using the variance requires that the returns be normally distributed. However, if stock returns follow a random walk and are IID (Independent, Identically distributed) random variables, then the Central Limit Theorem of calculus (or the Law of Large Numbers) states that the distribution would be normal and variance would be finite. Investors would thus desire the portfolio with the highest expected return for a given level of risk. Investors were expected to be risk-averse. This approach became known as mean/variance efficiency.

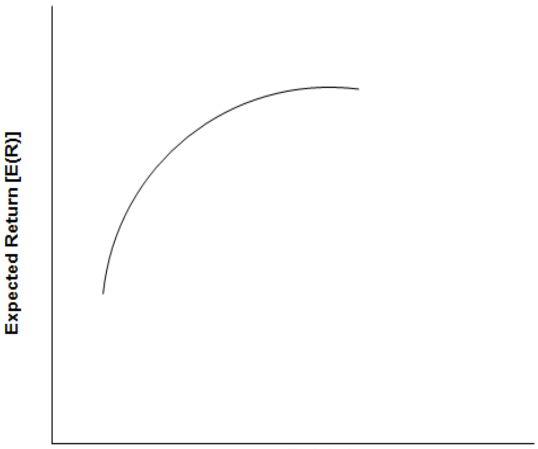



**Figure 1: The Efficient Frontier**

The curve shown in Figure 1 shows the highest level of expected return for a given level of risk or standard deviation. Investors would prefer these optimal portfolios, based on the rational investor model.

These concepts were extended by Sharpe (1964), Litner (1965), and Mossin (1966) in what came to be known as the Capital Asset Pricing Model (CAPM), the name coined by Sharpe. The CAPM combined the EMH and Markowitz mathematical model of portfolio theory into a model of investor behaviour based on rational expectations that is; they interpreted the information in the same manner. The CAPM was a remarkable advance. The CAPM begins by assuming that we live in a world free of transaction costs, commissions and taxes. Next, CAPM assumes that everyone can borrow and lend at a risk-free rate of interest, which is usually interpreted as the 90-day T-Bill rate. Finally, it assumes that all investors desire Markowitz mean/variance efficiency that they want the portfolio with the highest level of expected returns for a given level of risk and are risk-averse. Risk is again defined as the standard deviation of returns.

Based on these assumptions, the CAPM goes on to draw a number of conclusions about investor behaviour. First, the optimal portfolio for all investors would be combination of the market portfolio and the risk less asset. This type of portfolio is shown in Figure 2.

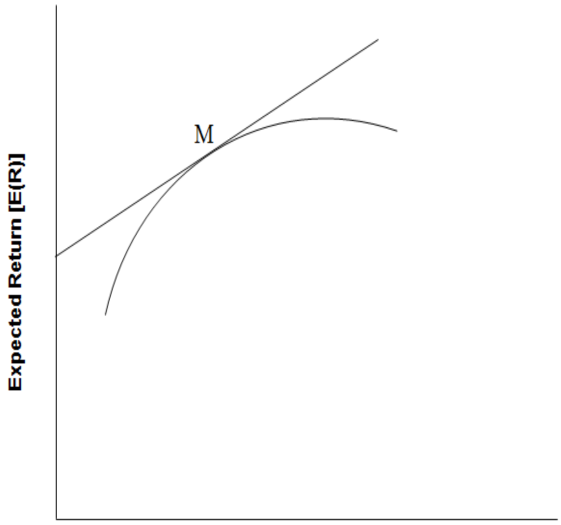
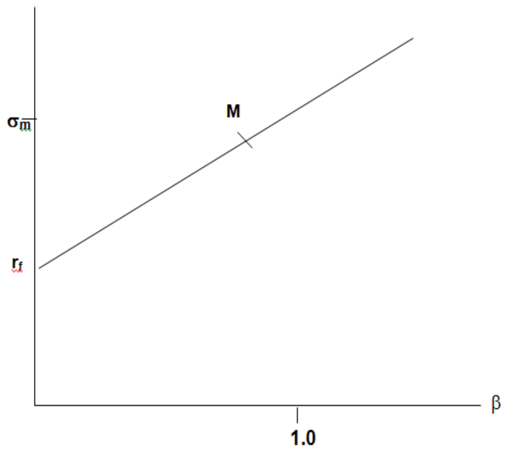



**Figure 2: The Capital Market Line**

In above figure a line is tangent to the efficient frontier at the market portfolio (M) and the Y-intercept, which is the risk – free rate(r). Levels of risk can be adjusted by adding to the risk less asset, to reduce the standard deviation of the portfolio. The portfolios that lie along this line, called the Capital Market Line (CML), dominate the portfolios on the efficient frontier; investors would prefer these portfolios to all others. In addition, investors are not compensated for assuming non market risk because the optimal portfolios are along the CML. The model also states that assets with higher risk should be compensated by higher returns. The risk is now relative to the market portfolio, a linear measure of the sensitivity of the security risk to the market risk is used. The linear measure is called beta. If all risky assets were plotted on a graph of their betas versus their expected returns, the result would be a straight line that intercepts the Y-axis at the risk – free rate of interest and passes through the market portfolio.

This result, called the Security Market Line (SML), as shown in Figure. 3.

**Figure 3: The Security Market Line**




The CAPM, which made quantitative methods practical, remains the standard for any new model of investor behaviour. Markowitz portfolio theory explained why diversification reduced risk. The CAPM explained how investors would behave, if they were rational. The CAPM underlying assumptions, which were simplifying assumptions, did not detract from the usefulness of the model. The merger of the EMH with CAPM and its modifications came to be generally known as Modern Portfolio Theory (MPT). This theory is based on a linear view of society. In this view people see information and adjust to it immediately and securities do so through their betas, which are the slope of a linear regression between a stock and the market portfolios excess returns. The linear paradigm is built into normality assumption.

Mathematically, this is a very clean model; however, it uses linear tools to model a non linear world. Much of our world involves non linear structures such as clouds, mountains and trees. Nature itself is non linear and rarely takes on a pattern of linear growth and development. In the past, it made sense for economists to use these linear methods of modeling and forecasting for financial markets: they were constrained by limitations in technology and vision. This lead early economists and mathematicians to rely on Euclidean objects as their tools of analysis: lines, planes etc. Hence, before the advent of computers, this oversimplification of market conditions was necessary to develop equations that could be solved by hand. When computers came along, however, it would become possible to model complex systems without relying on simplified assumptions. By the 1970s, non linear methods of analysis had been introduced in many fields of study including physics, biology, chemistry, electrical – engineering and sociology; however, although financial markets seemed to exhibit many non linear tendencies, they were not accepted into the financial investment community. On rationale for this resistance is that acceptance of a new paradigm would through out more than 50 years of work and such disruptions would completely change the way analysts conducted their business. Analysts' only recourses were to continuously introduce additional theories and variations of the EMH to explain market inconsistencies. The only problem was that the backbone of these new theories relied heavily on the correctness of the EMH. But the EMH was far from correct.

The development of the EMH was flawed from the beginning. First, a model was developed by making broad assumptions such as normal distributions of returns and the concept of a rational investor. Afterwards, the facts were presented in such a way so as to conform and thus support this model. This is just like putting the cart before the horse – it just does not work. The correct approach is to view the facts as they are, then find a theory which seems to fit them. From the evidence, the markets definitely take on many non linear characteristics which the EMH can not explain. These characteristics are as follows:-

1. People are not necessarily risk-averse at all times. They can often be risk-seeking, particularly if they are faced with what are perceived to be sure losses for not gambling.
2. People are biased when they set subjective probabilities. They are likely to be more confident in their forecasts than in warranted by the information they have.
3. People may not react to information as it is received. Instead, they may react to it after it is received, if it confirms a change in a recent trend. This is a non-linear reaction, as opposed to the linear reaction predicted by the rational investor concept.
4. There is no evidence to support the belief that people in aggregate are more rational than individuals.

**The Capital-Asset Pricing Model As A Logistic Equation**

The capital-asset pricing model equation is: **E(R) = $\alpha$ + $\beta$ E(Rm).** It says the expected return on a stock, **E(R)**, is proportional to the return on the market, **E(Rm)**. The input is **E(Rm)**. We multiply it by $\beta$ ("beta"), then add $\alpha$ ("alpha") to the result – to get the output **E(R)**. Chaotic systems are very sensitive to initial conditions. Suppose we have the following simple system (called a logistic equation) with a single variable, appearing as input, x(n), and output x(n+1). x(n+1) = a*x(n) (Buyer's effect) where a is the particular rate at which demand by buyer causes the price to rise. The seller's effect is that when prices increases at a*x(n), seller reduces the price at a*$x^2$(n). Therefore output is x(n+1) = a*x(n)-a*$x^2$(n); **x(n+1) = a*x(n)* [1-x(n)] where 0<x(n)≤1 and 0<a≤4** after that value of x(n+1) turns to be negative and prices can not be negative. Let x(n) = .75. The output is 4(.75) [1-.75] =.75. That is x(n+1) = .75. If this were an equation describing the price behaviour of a market, the market would be in equilibrium, because today's price (.75) would generate the same price tomorrow. If x(n) and x(n+1) were expectations, they would be self-fulfilling. Given today's price of x(n) = .75, tomorrow's price will be x(n+1) = .75. The value of .75 is called a **fixed point** of the equation, because using it as an input returns it as an output. It stays fixed, and doesn't get transformed into a new number. But, suppose the market starts out at x(0) = .7499. The output is **4(.7499) [1-.7499] = .7502 = x(1)**. Now using

the previous day's output x(1) = .7502 as the next input, we get as the new output: **4(.7502) [1-.7502] = .7496 = x(2)**. And so on. Going from one set of inputs to an output is called iteration**.** Then, in the next iteration, the new output value is used as the input value, to get another output value. Each set of solution paths – x(n), x(n+1), x(n+2), etc. – are called trajectories. The 100 iterations of the logistic equation, starting with x(0) = .7499 and x(0) = .74999 are shown in Table 1 (Annexure I). Clearly a small change in the initial starting value causes a large change in the outcome after a few steps. The equation is very sensitive to initial conditions.

Let ε denote the error in our initial observation, or the difference in two initial conditions. In Table 1, it could represent the difference between .75 and .7499 or between .75 and .74999. Let R be a distance (plus or minus) around a reference trajectory and then there is a question: how quickly does a second trajectory – which includes the error ε - get outside the range R? The answer is a function of the number of steps **n**, and the Lyapunov exponent λ(lemda), according to the following equation (where "exp" means the exponential **e** = 2.7182818…, the basis of the natural logarithms): **R = ε • exp(λ n).**

The Lyapunov exponent for an equation f(x(n)) is the average absolute value of the natural logarithm(log) of its derivatives: λ=Σ(1/n) log|df/dx(n)|

For example, the derivative of the right-hand side of the logistic equation x(n+1)=4 x(n) [1-x(n)]=4 x(n) – 4 $x^2$ (n) is 4-8 x(n). Thus for the first iteration of the second trajectory in Table 1, where x(n) = .7502, we have |df/dx(n)| = | 4[1-2(.7502)]| =2.0016 and log (2.0016) =.6939.If we sum over this and subsequent values, and take the average , we have the Lyapunov Exponent. Even we can start with x(0) = .1 and obtain the Lyapunov exponent. This is done in Table 2 (Annexure I).

In Table 2 only after ten iterations the empirically calculated Lyapunov exponent is .697226, near to its value of .6939.Thus the Lyapunov exponent of the logistic equation is λ = .697226. So in this instance, we have **R = ε • exp(.697226 n)**.

**Sample Calculations Using a Lyapunov Exponent**

In Table 1 we used starting values of .75, .7499 and .74999. That is, with a slightly different starting value, how many steps does it take before the system departs from the interval (.75, .76)? In this case the distance R = .01. For the second trajectory, with a starting value of .7499, the change in the initial condition is ε = .0001 (that is, ε = 75 - .7499). Hence, applying the equation R = ε • exp (λ n), we have **.01 = .0001 exp (.697226 n).** Solving for n, we get n = 6.64. We see that for n = 7 (the $7^{th}$ iteration), the value is x(7) = .762688, and that this is the first value that has gone outside the interval (.75, .76). Similarly, for the third trajectory, with a starting value of .74999, how many steps does it take before the system departs from the interval (.74, .75)? In this case the distance R =.01. Applying the equation R = ε • exp (λ n) yields **.01 = .00001 exp (.697226 n)**. Which solves to n = 9.96. We see that for n = 10 (the $10^{th}$ iteration), we have x(10) = .739691 and this is the first value outside the interval (.74, .75)for this trajectory. In this sample calculation, the system diverges because the Lyapunov exponent is positive. If it were the case the Lyapunov exponent were negative, λ <

0, then exp (λ n) would get smaller with each step. So it must be the case that λ > 0 for the system to be chaotic. The particular logistic equation, x(n+1) = 4 x(n) [1-x(n)], which we used is a simple equation with only one variable, namely x(n). So it has only one Lyapunov exponent. In general, a system with **M** variables may have as many as **M** Lyapunov exponents. In that case, an attractor is chaotic if at least one of its Lyapunov exponents is positive.

**Conclusion**

The earlier approaches to model the stock prices movements were using the linear models and accepting the normal distribution assumption. Of late, it is largely accepted that the assumption of normality and linearity are too weak to capture the intricacies in the movements of stock prices as in the case with most of the economic time series. The successful application of non- linear models by the peers of physical and natural sciences to explain several (seemingly random) phenomenons provoked the economists and financial analysts to assemble non linear models for stock market. What has been realized after many attempts is that several variables that determine stock prices are not independent, but are quite often dependent and the time when a particular variable starts moving in one direction or the other is uncertain. Moreover, there seems to be no regularity in changes in price due to change in other variables and price movements in the market resembles chaos. Furthermore, these patterns are not easily evident and are often masked by noise. The problem, therefore, is to find order in the chaotic nature of the stock market. Tests of fractals, chaos and other non-linear structures in financial markets analyze whether stock prices have some degree of auto dependency on their own past movements. In this context, auto dependency would mean that past stock prices, for example, could be used to determine the behavior of future stock prices. If a series has a certain degree of non-linear auto dependency, it will generate complex behavior. Precisely, the fractal and chaotic tests can describe it adequately. In short 'chaos' makes a neat connection with common experience. The gradual realization that chaos is relevant to such a wide range of subjects and provides a rich field for new research initiatives is certainly a factor in its recent establishment as one of the 'trendy' academic areas.

**References**

1. *Baumol, W.J., & Bnhabib, J. (1989). Chaos: Significance, Mechanism and Economic Applications. Journal of Economic Perspectives, 1.3, pp.77-105.*
2. *Bhattacharyya, Pranab Kumar (1998). Efficient Market Hypothesis and The Indian Securities Market in the Post-Reform Era. The Indian Journal of Commerce (Vol.51), Oct.-Dec.*
3. *Chaudhuri, S.K. (1991). Short-run Share Price Behaviour. New Evidence on Weak*

*Form of Market Efficiency. Vikalpa (Vol.16), 99.17-21.*
4. *Datero, M., & Madhusoodanan, T.P. (1997). Application of Fractals and Chaos Theory to the Indian Stock Market. Journal of Financial Management and Analysis, pp.26-42.*
5. *Peters, E.E. (1989). Fractal Structure in the Capital Markets. Financial Analysts Journal, July-August, pp.32-37.*
6. *Peters, E.E. (1991). A Chaotic Attractor for the S&P 500. Financial Analysts Journal, March-April, pp.55-62, 81.*
7. *Gleick, James (1987). Chaos: Making A New Science. New York. Penguin Books.*
8. *Srivastava, Puneet (1998). Chaos Theory. Vivek, pp.21-25.*
9. *Madapati, Ravi (2003). The Growing Importance of Chaos Theory. ICFAI Journal, Jan, pp.21-31.*
10. *Lucking, Richard (1991). Chaos– the origins and relevance of a new discipline, Project Appraisal. March, pp.23-32.*
11. *Fama, E.(1970). Efficient Capital Markets – A Review of Theory and Empirical Work. Journal of Finance (Vol.25), pp.383-417.*
12. *Gupta, O.P. (1992). Stock Market Efficiency: Anavaloues Price Behaviour – A Survey. Indian Journal of Finance and Research (Vol.2), pp.69-87.*
13. *Scheinkman, J.A. & LcBaron, B. (1989). Non-linear Dynamics and Stock Returns. Journal of Business. 62(3), pp.311-37.*
14. *Vidyanathan, R. and Kanti Kumar Gali (1994). Efficiency of Indian Capital Market. Indian Journal of Finance and Research (Vol.2), pp.27-40.*
15. *Wolf, A., Swift, J.B. & Vastano, J.A. (1985). Determining Lyapunov Exponents from a Time Series, Physica 16D.*

**Annexure I**

**Table 1: First One Hundred Iterations of the Equation x(n+1) = 4x(n) [1-x(n)] with different values of x(0)**

| **x(0)** | **0.75** | **0.7499** | **0.74999** |
|---|---|---|---|
| 1 | 0.75 | 0.7502 | 0.75002 |
| 2 | 0.75 | 0.7496 | 0.74996 |
| 3 | 0.75 | 0.7508 | 0.75008 |
| 4 | 0.75 | 0.748398 | 0.74984 |
| 5 | 0.75 | 0.753193 | 0.75032 |
| 6 | 0.75 | 0.743573 | 0.74936 |
| 7 | 0.75 | 0.762688 | 0.751279 |
| 8 | 0.75 | 0.72398 | 0.747436 |
| 9 | 0.75 | 0.799332 | 0.755102 |
| 10 | 0.75 | 0.641601 | 0.739691 |
| 11 | 0.75 | 0.919796 | 0.770193 |
| 12 | 0.75 | 0.295084 | 0.707984 |
| 13 | 0.75 | 0.832038 | 0.826971 |
| 14 | 0.75 | 0.559002 | 0.57236 |
| 15 | 0.75 | 0.986075 | 0.979056 |
| 16 | 0.75 | 0.054924 | 0.08202 |
| 17 | 0.75 | 0.207628 | 0.30117 |
| 18 | 0.75 | 0.658075 | 0.841867 |
| 19 | 0.75 | 0.900049 | 0.532507 |
| 20 | 0.75 | 0.359844 | 0.995773 |
| 21 | 0.75 | 0.921426 | 0.016836 |
| 22 | 0.75 | 0.289602 | 0.06621 |
| 23 | 0.75 | 0.82293 | 0.247305 |
| 24 | 0.75 | 0.582864 | 0.744581 |
| 25 | 0.75 | 0.972534 | 0.76072 |
| 26 | 0.75 | 0.106845 | 0.728099 |
| 27 | 0.75 | 0.381716 | 0.791883 |
| 28 | 0.75 | 0.944036 | 0.659218 |
| 29 | 0.75 | 0.211329 | 0.898598 |
| 30 | 0.75 | 0.666675 | 0.364478 |
| 31 | 0.75 | 0.888878 | 0.926535 |
| 32 | 0.75 | 0.395097 | 0.272271 |

| | | | |
|---|---|---|---|
| 33 | 0.75 | 0.955981 | 0.792559 |
| 34 | 0.75 | 0.168324 | 0.657638 |
| 35 | 0.75 | 0.559965 | 0.900602 |
| 36 | 0.75 | 0.985617 | 0.358074 |
| 37 | 0.75 | 0.056705 | 0.919428 |
| 38 | 0.75 | 0.213956 | 0.296322 |
| 39 | 0.75 | 0.672716 | 0.834061 |
| 40 | 0.75 | 0.880676 | 0.553614 |
| 41 | 0.75 | 0.420342 | 0.988502 |
| 42 | 0.75 | 0.974618 | 0.045463 |
| 43 | 0.75 | 0.09895 | 0.173583 |
| 44 | 0.75 | 0.356635 | 0.573809 |
| 45 | 0.75 | 0.917786 | 0.978209 |
| 46 | 0.75 | 0.301821 | 0.085265 |
| 47 | 0.75 | 0.8429 | 0.311979 |
| 48 | 0.75 | 0.529679 | 0.858592 |
| 49 | 0.75 | 0.996477 | 0.485646 |
| 50 | 0.75 | 0.014044 | 0.999176 |
| 51 | 0.75 | 0.055386 | 0.003294 |
| 52 | 0.75 | 0.209273 | 0.013132 |
| 53 | 0.75 | 0.661911 | 0.051837 |
| 54 | 0.75 | 0.895139 | 0.196601 |
| 55 | 0.75 | 0.375462 | 0.631796 |
| 56 | 0.75 | 0.937961 | 0.930519 |
| 57 | 0.75 | 0.232761 | 0.258613 |
| 58 | 0.75 | 0.714334 | 0.76693 |
| 59 | 0.75 | 0.816243 | 0.714994 |
| 60 | 0.75 | 0.599961 | 0.81511 |
| 61 | 0.75 | 0.960031 | 0.602824 |
| 62 | 0.75 | 0.153485 | 0.957709 |
| 63 | 0.75 | 0.51971 | 0.162009 |
| 64 | 0.75 | 0.998446 | 0.543049 |
| 65 | 0.75 | 0.006206 | 0.992587 |
| 66 | 0.75 | 0.02467 | 0.029431 |
| 67 | 0.75 | 0.096247 | 0.114261 |
| 68 | 0.75 | 0.347933 | 0.404822 |
| 69 | 0.75 | 0.907503 | 0.963765 |
| 70 | 0.75 | 0.335767 | 0.139689 |
| 71 | 0.75 | 0.89211 | 0.480705 |
| 72 | 0.75 | 0.385 | 0.998511 |
| 73 | 0.75 | 0.9471 | 0.005948 |
| 74 | 0.75 | 0.200407 | 0.02365 |
| 75 | 0.75 | 0.640977 | 0.092363 |
| 76 | 0.75 | 0.920502 | 0.335328 |
| 77 | 0.75 | 0.292712 | 0.891533 |
| 78 | 0.75 | 0.828127 | 0.386809 |
| 79 | 0.75 | 0.56933 | 0.948751 |
| 80 | 0.75 | 0.980773 | 0.19449 |
| 81 | 0.75 | 0.075428 | 0.626655 |
| 82 | 0.75 | 0.278955 | 0.935834 |

| | | | |
|---|---|---|---|
| 83 | 0.75 | 0.804557 | 0.240196 |
| 84 | 0.75 | 0.62898 | 0.730008 |
| 85 | 0.75 | 0.933457 | 0.788386 |
| 86 | 0.75 | 0.24846 | 0.667334 |
| 87 | 0.75 | 0.74691 | 0.887998 |
| 88 | 0.75 | 0.756142 | 0.397831 |
| 89 | 0.75 | 0.737565 | 0.958246 |
| 90 | 0.75 | 0.774252 | 0.160042 |
| 91 | 0.75 | 0.699143 | 0.537713 |
| 92 | 0.75 | 0.841369 | 0.994311 |
| 93 | 0.75 | 0.533869 | 0.022627 |
| 94 | 0.75 | 0.995411 | 0.08846 |
| 95 | 0.75 | 0.01827 | 0.32254 |
| 96 | 0.75 | 0.071744 | 0.874032 |
| 97 | 0.75 | 0.266388 | 0.440402 |
| 98 | 0.75 | 0.781702 | 0.985792 |
| 99 | 0.75 | 0.682577 | 0.056024 |
| 100 | 0.75 | 0.866663 | 0.211541 |

**Table 2: Empirical Calculation of Lyapunov Exponent from the Logistic Equation with x(0) = .1,**

| Iteration | X(n) = .1 | df/dx(n) | log\|df/dx(n)\| |
|---|---|---|---|
| 1 | 0.36 | 1.12 | 0.113329 |
| 2 | 0.9216 | -3.3728 | 1.215743 |
| 3 | 0.289014 | 1.687890 | 0.523479 |
| 4 | 0.821939 | -2.575514 | 0.946049 |
| 5 | 0.585421 | -0.683364 | -0.380728 |
| 6 | 0.970813 | -3.766507 | 1.326148 |
| 7 | 0.113339 | 3.093286 | 1.129234 |
| 8 | 0.401974 | 0.784209 | -0.243079 |
| 9 | 0.961563 | -3.692508 | 1.306306 |
| 10 | 0.147837 | 2.817308 | 1.035782 |
| | | | |
| | | | |
| | Average | | 0.697226 |