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Analysis of A Two-Unit Chargeable Standby System with Interchangeable Units and Correlatedfailures and Repairs

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Abstract

This paper presents the analysis of a two-unit chargeable standby system with interchangeable units(identical) i.e. the operative and standby units are interchanged at random epochs. The failure and repair times of each unit are assumed to be correlated and their joint density is taken as bivariate exponential. Using regenerative point technique, various reliability characteristics of the system have been obtained. The behavior of MTSF has also been studied through graph.

Keywords: Chargeable Standby Unit, Interchangeable Units, MTSF

Introduction

Various researchers including references[1, 2] have analysed two-unit standby system assuming that standby unit becomes operative whenever operative unit fails and also failure and repair times are uncorrelated. Sometimes we come across the systems where operative unit requires some rest and standby units needs its operation i.e. operative and standby units are interchanged at random epochs. Such standbys are known as chargeable standby. Ref. [3] has analysed a two-unit chargeable standby system with interchangeable units. But in practical life, it is also observed that there are some systems[4, 5] with correlated failure and repair times.

Keeping these factors in view, we analyse here a two-unit chargeable standby system with interchangeable units and correlated failure and repair times.

Model Description

The system consists of two identical units. Initially one unit is operative and the other is chargeable standby. The operative and standby units are interchanged at random epochs. The chargeable standby may be found charged with probability p and uncharged with probability q at the time of need. A single repair facility is available to repair a failed unit and also to charge an uncharged unit. When system is down, the priority is given to the charging of an uncharged unit. The joint distribution of failure and repair times of each unit is bivariate exponential. If the operative unit fails while the other unit is under repair, the unit which failed later is taken up for repair superseding the repair of the unit already in hand. The residual repair time of a unit whose repair is interrupted need not depends on its failure time. The distribution of residual repair time (y´) is negative exponential with parameter θ. A repaired unit works like a new one.

Notation And States

In order to define the states of the system, we define the following symbols.

N_o/N_s unit is normal mode and operative/ chargeable standby

N_c/N_{cc} uncharged unit is under the process of charging/ charging is continued from earlier state

$F_r/F_w/F_{ir}/F_{iw}$ failed unit is under repair/ waiting for repair/again taken up for repair after interruption/waiting for repair after interruption

Now, using the above symbols, we have following states of system.

up states:

$\underline{S}_0=(N_o, N_s)$, $S_1=(N_o, N_c)$, $S_2=(F_r, N_o)$,$\underline{S}_5=(F_{ir}, N_o)$

Down states:

$\underline{S}_3= (F_w, N_c)$, $S_4= (F_{iw}, F_r)$, $S_6= (F_w, N_{cc})$

The underlined states are regenerative. Transition diagram is shown in fig.1. The other symbols, used in the paper are defined as follows.

X,Y failure and repair times of an operative and failed unit

f(x, y) joint density of X and Y, i.e.

$f(x, y)= \alpha\beta(1-r)\, e^{-\alpha x-\beta y} I_0[2\sqrt{(\alpha\beta xy)}]$ g(x)=α
g(x) marginal density of X, i.e.
$(1-r)e^{-\alpha(1-r)x}$ h(y) marginal density of Y, i.e. $h(y)=\beta(1-r)e^{-\beta(1-r)y}$

y´ residual repair time of a unit after interruption of repair
μ constant interchangeable rate between operative and chargeable standby unit
λ constant charging rate of an uncharged unit
p P[chargeable standby unit is found to be charged at time of need]

Transition Probability And Sojourn Times

We obtain the following non-zero elements of transition probability matrix P.

$p_{00}=p\mu A^{-1}$, $p_{03}=q\alpha(1-r)A^{-1}$
$p_{00}^{(1)}=q\mu\lambda A^{-1}B^{-1}$, $p_{05}^{(1,6,2,4)}=q\mu\alpha^2(1-r)A^{-1}B^{-1}C^{-1}$
$p_{00}^{(1,6,2)}=q\mu\alpha\beta(1-r)A^{-1}B^{-1}C^{-1}$
$p_{00}^{(2)}=p\alpha\beta(1-r)A^{-1}C^{-1}$, $p_{05}^{(2,4)}=p\alpha^2(1-r)A^{-1}C^{-1}$
$p_{30}^{(2)}=\beta C^{-1}$, $p_{35}^{(2,4)}=\alpha C^{-1}$
$p_{50}=\theta D^{-1}$ and $p_{55}^{(4)}=\alpha(1-r)D^{-1}$

where
$A=[\mu+\alpha(1-r)]$, $B=[\lambda+\alpha(1-r)]$, $C=(\alpha+\beta)$ and $D=[\theta+\alpha(1-r)]$
Clearly, these probabilities satisfy the following relations:
$p_{00}+p_{00}^{(1)}+p_{00}^{(1,6,2)}+p_{05}^{(1,6,2,4)}+p_{03}+p_{00}^{(2)}+p_{05}^{(2,4)}=1$, $p_{30}^{(2)}+p_{35}^{(2,4)}=1$ and $p_{50}+p_{55}^{(4)}=1$
The mean sojourn times in the various states are:
$T_0=A^{-1}$, $T_1=B^{-1}$, $T_2=[(1-r)C]^{-1}$, $T_3=T_6=\lambda^{-1}$, $T_4=[\beta(1-r)]^{-1}$ and $T_5=D^{-1}$

MEAN TIME TO SYSTEM FAILURE

Considering the failed states S_3, S_4 and S_6 as absorbing, we have, by simple probabilistic reasoning, the following relations.

$$\pi_0(t)=Q_{00}(t)\,Ⓢ\,\pi_0(t)+Q_{03}(t)+Q_{03}(t)+Q_{00}^{(1)}(t)\,Ⓢ\,\pi_0(t)+Q_{06}^{(1)}(t)+Q_{00}^{(2)}(t)\,Ⓢ\,\pi_0(t)+Q_{04}^{(2)}(t) \text{ ------(1)}$$

Taking the Laplace-Stieltjes of (1) and using the well-known formula for MTSF, we get

$$MTSF=(T_0+p_{01}T_1+p_{02}T_2)/(1-p_{00}-p_{00}^{(1)}-p_{00}^{(2)}) \text{ ---(2)}$$

AVAILABILITY ANALYSIS

Let $A_i(t)$ be the probability that the system is up at epoch t when the system initially starts from regenerative state S_i. Now using the arguments of the theory of regenerative processes, we observe:

$$A_0(t)=M_0(t)+q_{00}(t)©A_0(t)+q_{03}(t)©A_3(t)+q_{00}^{(1)}(t)©A_0(t)+q_{00}^{(2)}(t)©A_0(t)+q_{00}^{(1,6,2)}(t)©A_0(t)+q_{05}^{(1,6,2,4)}(t)©A_5(t)+q_{05}^{(2,4)}(t)©A_5(t) \text{ ---(3)}$$

$$A_3(t)=q_{30}^{(2)}(t)©A_0(t)+q_{35}^{(2,4)}(t)©A_5(t) \text{ ---(4)}$$

$$A_5(t)=M_5(t)+q_{50}(t)©A_0(t)+q_{55}^{(4)}(t)©A_5(t) \text{ ---(5)}$$

Where
$M_0(t)=e^{-At}+q_{o1}(t)©e^{-Bt}+q_{02}(t)©e^{-C(1-r)t}$
$M_5(t)=e^{-Dt}$
By taking the Laplace transform of equations (3)-(5) we can obtain $A_0^*(s)$. Using this result, the steady state availability of the system is given by:

$$A_0=\lim_{t\to\infty} A_0(t)=\lim_{s\to 0} s\,A_0^*(s)=N_1\,D^{-1} \text{ ---(6)}$$

where
$N_1=(T_0+p_{01}T_1+p_{02}T_2)p_{50}+T_5\, p_{03}\, p_{55}^{(2,4)}$
and
$D=(m_{00}+m_{00}^{(1)}+m_{00}^{(2)}+m_{00}^{(1,6,2)}+m_{05}^{(1,6,2,4)}+m_{05}^{(2,4)}+m_{03})p_{50}+(m_{30}^{(2)}+m_{35}^{(2,4)})p_{03}p_{50}+(m_{50}+m_{55}^{(4)})(p_{05}^{(1,6,2,4)}+p_{05}^{(2,4)}+p_{03}p_{35}^{(2,4)})$

3

Busy Period Analysis

Let $B_i(t)$ be the probability the repairman is busy at epoch t when system initially starts from regenerative state Si. By simple probabilistic reasoning we have

$B_0(t)=q_{00}(t)©B_0(t)+q_{03}(t)©B_3(t)+q_{00}^{(1)}(t)©B_0(t)+q_{00}^{(2)}(t)©B_0(t)+q_{00}^{(1,6,2)}(t)©B_0(t)$
$+q_{05}^{(1,6,2,4)}(t)©B_5(t)+q_{05}^{(2,4)}(t)©B_5(t)$ ---(7)

$B_3(t)=W_3(t)+q_{30}^{(2)}(t)©B_0(t)+q_{35}^{(2,4)}(t)©B_5(t)$ ---(8)

$B_5(t)=W_5(t)+q_{50}(t)©B_0(t)+q_{55}^{(4)}(t)©B_5(t)$ ---(9)

where

$W_3(t)=e^{-\lambda t}+q_{32}(t)©e^{-C(1-r)t}+q_{34}^{(2)}(t)©e^{-\beta(1-r)t}$

$W_5(t)=e^{-Dt}+q_{54}(t)©e^{-\beta(1-r)t}$

By taking the Laplace transform of equation (7) to (9) we can get $B_0^*(s)$ and the steady-state probability that the repairman is busy is given by

$B_0=\lim_{t\to\infty} B_0(t)=\lim_{s\to\infty} sB_0^*(s)=N_2D^{-1}$ ---(10)

where

$N_2=(T_2+T_3+p_{24}T_4)p_{03}p_{50}+(T_4+T_5)$
$(p_{03}\, p_{35}^{(2,4)}+p_{05}^{(1,6,2,4)}+p_{05}^{(2,4)})$

Cost Analysis

The expected up-time of the system in (0, t] is

$\mu_{up}(t)=\int_0^t A_0\,(u)\,du$

so that

$\mu_{up}^*(s)=A_0^*(s)\,/\,s$ ---(11)

The expect duration of the busy period of the repairman in (0, t] is

$\mu_b(t)=\int_0^t B_b\,(u)\,du$

so that

$\mu_b^*(s)=B_0^*(s)\,/\,s$ ---(12)

Now that expected profit incurred in (0, t] is

$P(t)=k_1\,\mu_{up}(t)-k_2\,\mu_b(t)$ ---(13)

Where k_1 revenue per unit up-time and k_2 is the repair cost per unit time for which the repairman is busy. Therefore, expected profit per unit time in steady state is $P_0=\lim_{t\to\infty} P(t)/t=\lim_{s\to 0} s^2\, P_0^*(s)=k_1A_0-k_2B_0$---(14)

Conclusion

To observe the effect of correlation(r) and interchangeable rate (μ) on the system performance, we plot the MTSF with respect to r and μ keeping the other parameters fixed. From the fig 2 we observed that the MTSF increases with increasing r and μ. Thus we conclude that the system leads to a better overall performance for large value of correlation and interchangeable rate.

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TRANSITION DIAGRAM

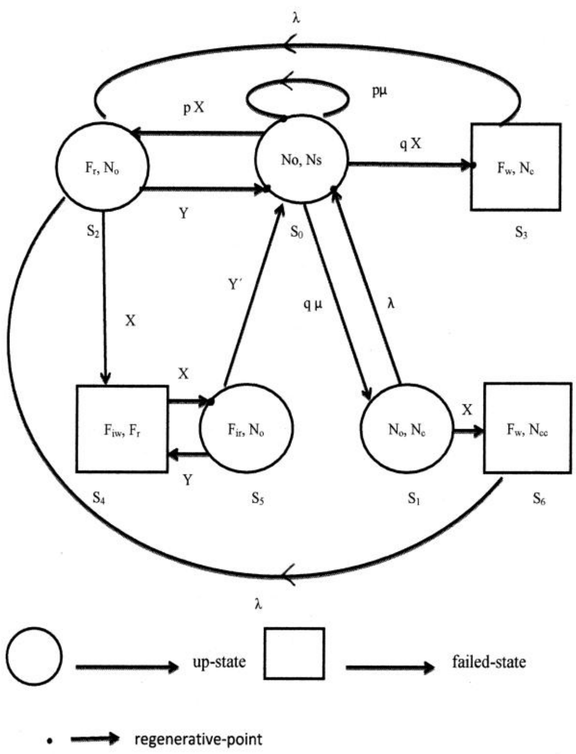

Fig.1

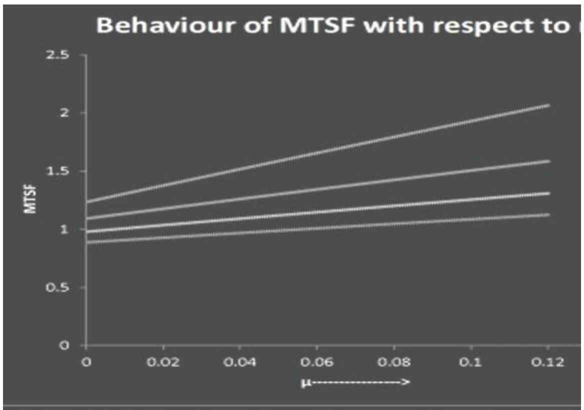

Fig.2