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Propagation of Correlations in Mhd Dusty Turbulence in A Rotating System

Abstract

In this paper we have derived the spectrum equation for the joint propagation of correlations, velocity and magnetic field. Various discussed the problem of very strong uniform magnetic field and rotating system of spectral equation.

Keywords: Propagation of Correlations, Hydromagnetic Turbulence, Continuity Equation, Uniform Magnetic Field, Magnetic Reynolds Number.

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Introduction

In real fluids, the viscous stresses in turbulent motions will cause the kinetic energy of the motions to dissipate in heat. If there are no external effects present to supply energy continuously for maintaining the turbulent motions, these will decay in the course of time. An interesting problem to investigate is how the flow pattern and the relations between velocities change during decay. Since these relations can be described by the tensor (Q_{ij}) A, B of the double velocity correlations, we have to consider the change in this tensor with time. Batchelor (1951) obtained an expression for the velocity, covariance between the fluctuating velocities at two different points, a distance r apart in a field of homogeneous isotropic turbulence. Jain (1962) using Chandrasekhar's (1955) new theory of turbulence, derived expressions for pressure and acceleration covariance in ordinary turbulence. A good deal of theoretical studies on magneto hydrodynamic turbulence has been made during last fifteen years. Some authors (see for instance, Ohji, 1964) considered MHD turbulence in the absence of an external magnetic field in order to gain a basic understanding of a self-adjusting process of the mechanical and magnetic modes of turbulence. In a variety of astrophysical and geophysical problems, however, it is often the case that a certain magnetic field such as the cosmic magnetic field, the geomagnetic field, etc. is imposed on a turbulent motion of a conducting fluid. The essential effect of the presence of an imposed magnetic field is that the mechanical and magnetic modes of turbulence interact not only with each other through the self-adjusting processes but also with the external magnetic field. If the external magnetic field is very strong, the effect of the latter interaction will predominate that of the self-adjusting processes. Ohji (1964) presented a first order theory for turbulence of an electrically conducting fluid in the presence of a uniform magnetic field, which is so strong that the nonlinear mechanism as well as the dissipation when compared with the external coupling terms are of minor importance. He discussed the effect of a very strong uniform magnetic field on incompressible MHD turbulence in the presence of a constant angular velocity and Hall effect.

Saffman (1962) observed the effect of dust particles on the stability of laminar flow of an incompressible fluid with constant mass concentration of dust particles. He has given the equations describing the motion of a fluid containing small dust particles. Using the equations given by Saffman, Michael and Miller (1966) has discussed the motion of dusty gas occupying the semi-infinite space above a rigid plane boundary. The behaviour of discrete particles in a turbulent flow is of great interest to many branches of technology, particularly if there is a substantial difference in density between the particles and the fluid. The combined flow of solids and fluids or of atomized liquids and gases (flow of mist) is encountered for instance in one or more of the technical applications like gas and liquid cleaners (e.g. cyclone separator), pneumatic conveying, coal washing, and mineral dressing, chemical reactors based on the fluidized solids system. The behaviour of dust particles in a turbulent fluid depends largely.

1. On the concentration of the particles
2. On the size of the particles with respect to the scale of turbulence fluid.

At great concentration there is interaction between the particles through collisions and through effects on the flow of the fluid in the neighbourhood of the particles. At extremely high concentrations near that of maximum packing of the particles the turbulence may be even ‘frozen’ a term introduced by Bagnold (1954) to denote a condition of almost entirely damped turbulent motion. When the concentration is very low we may neglect interference of the particles and regard each particle being alone in the turbulent flow field. Then has made first extensive theoretical study on the motion of a very small particle suspended in a turbulent fluid.

In this topic we have derived the expressions for velocity covariance and solution has been obtained in terms of defining scalars. Further the effect of very strong uniform magnetic field is discussed. It has been shown that the solutions of the spectrum equations so obtained are of oscillatory nature.

Basic Equations

The equations of motion and of continuity of an incompressible viscous dusty fluid are given by

$$\frac{\partial u_i}{\partial t}+\frac{\partial}{\partial x_k}(u_i u_k - h_i h_k) = -\frac{\partial w}{\partial X_i}+\nu\nabla^2 u_i + f(u_i - v_i) - 2\varepsilon_{nmi}\,\Omega_n u_i \quad (1.1)$$

$$\frac{\partial h_i}{\partial t}+\frac{\partial}{\partial X_k}(h_i u_k - u_i h_k) = \lambda\nabla^2 h_i \quad (1.2)$$

$$\frac{\partial v_i}{\partial t}+v_k\frac{\partial v_i}{\partial X_k} = -\frac{k}{m_s}(u_i - v_i) \quad (1.3)$$

$$\frac{\partial h_i}{\partial X_i} = \frac{\partial u_i}{\partial X_i} = \frac{\partial v_i}{\partial X_i} = 0 \quad (1.4)$$

where $m_s = 4/3\ \pi R_s^3 \rho_s$ is the mass of a single spherical dust particle of radius $R_s; k \pm 6\pi R_s \rho_v$ by the Stoke’s drag formula; $f = kN/\rho$ has dimensions of frequency; N = constant number density of dust particle; ρ_s = constant density of the material in dust particles; v_i (x t) are dust velocity component; (u_i (x, t)), are the fluid velocity components.

2. Discussion of the Problem

Let u_i denote the component of velocity at point x_i' time t' then

$$\frac{\partial u_i'}{\partial t'}+\frac{\partial}{\partial X_k'}\left(u_i' u_k' - h_i' h_k'\right) = -\frac{\partial w'}{\partial x_i'}+\nu\nabla_{x'}^2 u_i' + f\left(u_i' - v_i'\right) - 2\varepsilon_{nki}\Omega_k u_i' \quad (2.1)$$

Similarly if u_j'' at time t'' then

$$\frac{\partial u_j''}{\partial t''}+\frac{\partial}{\partial X_k''}\left(u_i'' u_k'' - h_i'' h_k''\right) = -\frac{\partial w''}{\partial x_j''}+\nu\nabla_{x''}^2 u_j'' + f\left(u_j'' - v_j''\right) - 2\varepsilon_{pkj}\Omega_p u_j'' \quad (2.2)$$

Multiplying (2.1) by u_j'' and (2.2) by u_i' and averaging the resulting equation we have

$$\frac{\partial}{\partial t}\overline{u_i' u_j''}+\frac{\partial}{\partial x_k}(\overline{u_i' u_k' u_j''} - \overline{h_i' h_k' u_j''})+\frac{\partial}{\partial x_{k'}}(\overline{u_i'' u_k'' u_i'} - \overline{h_i'' h_k'' u_i'})$$
$$= -\frac{\partial}{\partial X_{j''}}\overline{w'' u_i'} - \frac{\partial}{\partial x_i'}\overline{w' u_j''} + \nu\nabla_{x''}^2\overline{u_i' u_j''} + \nu\nabla_{x'}^2\overline{u_i' u_j''} + f(\overline{u_i' u_j''} - \overline{v_i' u_j''}$$
$$+\overline{u_j'' u_i'} - \overline{v_j'' u_i'}) - 2\varepsilon_{nki}\Omega_k\overline{u_i' u_j''} - 2\varepsilon_{pkj}\Omega_p\overline{u_i' u_j''} \quad (2.3)$$

We assume that the particle are non conducting and therefore $\overline{h_i' v_j''} = \overline{h_j'' v_i'} = 0$. It is also assumed that the instantaneous velocities at one point remain unaffected by the dust particles of the other point i.e.

$$\overline{u_i' v_j''} = \overline{u_j'' v_i'} = 0$$

taking the average and using the condition of homogeneity we have

$$\frac{\partial}{\partial t}\overline{u_i' u_j''} - 2\frac{\partial}{\partial \varepsilon_k}(\overline{u_i' u_k' u_j''} - \overline{u_i' n_k u_j''}) = -\frac{\partial}{\partial \varepsilon_j}\overline{w'' u_i'}$$
$$+\frac{\partial}{\partial \xi_i}\overline{w' u_j''} + 2\nu\nabla^2\overline{u_i' u_j''} + 2f(\overline{u_i' u_j''}) - 2(\varepsilon_{nki}\Omega_n + \varepsilon_{pkj}\Omega_p)\overline{u_i' u_j''} \quad (2.4)$$

where,

$$\xi_i = X_i'' - X_i' \quad \frac{\partial}{\partial \xi_i} = \frac{\partial}{\partial X_i''} = -\frac{\partial}{\partial X_i'}$$

and $\nabla^2 x' = \nabla_{x''}^2 = \nabla^2$

In deriving this equation (2.4) we have made the use of relations

$$\overline{u_i' u_j'' u_k''} = -\overline{u_i'' u_j'' u_k'} \text{ and } \overline{h_i' h_j'' u_k''} = -\overline{h_i'' h_j'' u_k'}$$

As suggested by Chandrasekhar (1955) we neglect the terms

$$\overline{w''\ u_i'} \quad \text{and} \quad \overline{w'\ u_j''}$$

We put

$$Q_{ij} = \overline{u_i'\ u_j''}$$
$$S_{ikj} = \overline{u_i' u_j'' u_k''}$$
$$L_{ikj} = \overline{h_i' h_j' u_j''}$$

The equation (2.4) becomes

$$\frac{\partial}{\partial t}\overline{u_i' u_j''} - 2\frac{\partial}{\partial \xi_k}\left(S_{ikj} - L_{ikj}\right)$$
$$= 2\nu\nabla^2 Q_{ij} + f\,Q_{ij} - 2(\epsilon_{nki}\,\Omega_n + \epsilon_{pkj}\,\Omega_p)\,Q_{ij} \quad (2.5)$$

We have (cf Dixit, 1989)

$$Q_{ij} = \frac{Q'}{r}\xi_i\,\xi_j - (rQ' + 2Q)\,\delta_{ij}$$

Also

$$\frac{\partial}{\partial \xi_k}S_{i.k.j} = \left[\frac{6}{r}S' + S''\right]\xi_i\xi_j - \left[10S + 8rs' + r^2 S''\right]\delta_{ij}$$

$$\frac{\partial}{\partial \xi_k}L_{i.k.j} = \left[\frac{6}{r}L' + L''\right]\xi_i\xi_j - \left[10L + 8rL' + + r^2 L''\right]\delta_{ij}$$

and

$$\nabla^2 Q_{ij} = \xi_i\xi_i\left[\frac{Q'''}{r} + \frac{4}{r^2}Q'' - \frac{4}{r^3}Q'\right] - \delta_{ij}\left[rQ''' + 6Q'' + \frac{4}{r}Q'\right]$$

Putting these in equation (2.5) we get

$$\frac{\partial}{\partial t}\overline{u_i' u_j''} = 2\left[\frac{6}{r}S' + S''\right]\xi_i\xi_j - 2\left[10S + 8rS' + r^2 S''\right]\delta_{ij}$$
$$-2\left[\frac{6}{r}L' + L''\right]\xi_i\xi_j + 2\left[10L + 8rL' + r^2 L''\right]\delta_{ij}$$
$$+2\nu\left[\frac{Q'''}{r} + \frac{4}{r^2}Q'' - \frac{4}{r^3}Q'\right]\xi_i\xi_j - 2\nu\delta_{ij}\left[rQ''' + 6Q'' + \frac{4}{r}Q'\right]$$
$$+f\left[\left(\frac{Q'}{r}\xi_i\xi_i\right) - (rQ' + 2Q)\right]\delta_{ij} - 2(\epsilon_{nki}\,\Omega_n + \epsilon_{pkj}\,\Omega_p)$$
$$\left(\frac{Q'}{r}\xi_i\xi_j - (rQ' + 2Q)\,\delta_{ij}\right)$$

or

$$\frac{\partial}{\partial t}\overline{u_i' u_j''} = \left[\frac{12}{r}(S' - L') + 2(S'' - L'') + 2\left(\frac{Q'''}{r} + \frac{4}{r^2}Q'' - \frac{4}{r^3}Q'\right) + f\,\frac{Q'}{r}\right.$$
$$\left.-2\left(\epsilon_{nki}\,\Omega_k + \Omega_k + \epsilon_{pkj}\,\Omega_p\right)\frac{Q'}{r}\right]\xi_i\xi_j$$
$$-\left[20(S - L) + 16r\,(S' - L') + 2r^2(S'' - L'') + 2\nu(rQ'''\right.$$
$$\left. + 6Q'' + \frac{4}{r}Q') + f(rQ' + 2Q)\right.$$

$$+2(\in_{nki}\Omega_k+\in_{pkj}\Omega_p)(rQ'+2Q)]\delta_{ij} \qquad (2.6)$$

$\overline{u_i' u_j'}$ can be put in the form

$$\overline{u_i' u_j''} = \alpha(r,t)\xi_i\xi_j + \beta(r,t)\delta_{ij}$$

Therefore, we have

$$\frac{\partial\alpha}{\partial t} = \frac{12}{r}(S'-L') + 2(S''-L'') + 2\left(\frac{Q'''}{r} + \frac{4}{r}Q'' - \frac{4}{r^3}Q'\right) + f\,\frac{Q'}{r}$$

$$-2\left(\in_{nki}\Omega_n + \varepsilon_{pkj}\Omega_p\right)\frac{Q'}{r} \qquad (2.7)$$

$$\frac{\partial B}{\partial t} - [20(S-L) + 16r(S'-L') + 2r^2(S''-L'')$$

$$+2\nu(rQ'''+6Q''+\frac{4}{r}Q') - f(rQ'+2Q)]$$

$$+2(\in_{nki}\Omega_n + \in_{pkj}\Omega_p)(rQ'+2Q) \qquad (2.8)$$

Equations (2.7) and (2.8) give the expressions for rate of change of defining scalars of velocity covariance in terms of defining scalars in presence of corriolis force.

The Effect of a Very Strong Magnetic Field

If a uniform strong magnetic field be imposed then the equations of motion can be written Ohji (1964) as:

$$\frac{\partial}{\partial t}\overline{u_i' u_j''} + H_k\frac{\partial}{\partial \xi_k}\left(\overline{h_i' u_j''} - \overline{u_i' h_j''}\right)$$

$$= \frac{\partial}{\partial \xi_k}\left(\overline{u_i' u_k' u_j''} - \overline{u_i' u_k'' u_j''} + \overline{u_i' u_k'' h_j''} - \overline{h_i' h_k' u_j''}\right)$$

$$+\frac{1}{\rho}\left(\frac{\partial \overline{P_*' u_j''}}{\partial \xi_i} - \frac{\partial \overline{u_i' P_*''}}{\partial \xi_j}\right) + 2\nu\nabla^2\,\overline{u_i' u_j''}$$

$$+f\left(\overline{u_i' u_j''} - \overline{v_i' u_j''} + \overline{u_j'' u_i'} - \overline{v_j'' u_i'}\right)$$

$$-2\in_{nki}\Omega_n \overline{u_i' u_j''} - 2\in_{pkj}\Omega_p \overline{u_i' u_j''} \qquad (3.1)$$

$$\frac{\partial}{\partial t}\overline{h_i' h_j''} + H_k\frac{\partial}{\partial \xi_k}\left(\overline{u_i' h_j''} - \overline{h_i' u_j''}\right)$$

$$= \frac{\partial}{\partial \xi_k}\left(\overline{h_i' u_k' u_j''} - \overline{h_i' u_k'' u_j''} + \overline{h_i' h_k' u_j''} - \overline{u_i' h_k' h_j''}\right)$$

$$+2\lambda\nabla^2\,\overline{h_i' h_j''} \qquad (3.2)$$

$$\frac{\partial}{\partial t}\overline{u_i' h_j''} + H_k\frac{\partial}{\partial \xi_k}\left(\overline{h_i' h_j''} - \overline{u_i' u_j''}\right)$$

$$= \frac{\partial}{\partial \xi_k}\left(\overline{u_i' u_k' h_j''} - \overline{u_i' u_k'' h_j''} + \overline{u_i' h_k'' h_j''} - \overline{h_i' h_k' h_j''}\right)$$

$$+\frac{1}{\rho}\frac{\partial \overline{p_*'' h_j''}}{\partial \xi_i} + (\nu+\lambda)\nabla^2\,\overline{u_i' h_j''} + f\left(\overline{u_i' h_j''} - \overline{v_i' h_j''}\right)$$

$$-2\in_{nki}\Omega_n\,\overline{u_i' h_j''} \qquad (3.3)$$

$$\frac{\partial}{\partial t}\overline{h_i' u_j''} + H_k\frac{\partial}{\partial \xi_k}\left(\overline{u_i' u_j''} - \overline{u_i' h_j''}\right)$$

$$= \frac{\partial}{\partial \xi_k}\left(\overline{h_i' u_k' u_j''} - \overline{u_i' h_k'' u_j''} + \overline{h_i' h_k'' h_j''} - \overline{h_i' u_k' u_j''}\right)$$

$$+\frac{1}{\rho}\frac{\partial \overline{h_i' p_*'}}{\partial \xi_i} + (\nu+\lambda)\nabla^2\,\overline{h_i' u_j''}$$

$$+f\left(\overline{h_i' u_j''} - \overline{h_i' v_j''}\right) - 2\in_{pkj}\Omega_p\,\overline{h_i' u_j''} \qquad (3.4)$$

where $p_* = p + \frac{\rho}{2}\left(h^2 + 2H_k h_k\right)$

In order to estimate the order of magnitude of various terms in the correlation equation obtained above, we introduce the characteristic length l and the level of turbulence

$a = \left[\frac{u^2+h^2}{3}\right]^{1/2}$, we have then (cf. Ohji, 1964).

$$\frac{\text{triple correlation terms}}{\text{External coupling term}} \sim \frac{a^3/1}{Ha^2/1} = \frac{a}{H} = \in$$

$$\frac{\text{viscous dissipation terms}}{\text{External coupling terms}} \sim \frac{\nu a^2/1^2}{Ha^2/1^2} = \frac{\nu}{H1} = \frac{1}{R_H}$$

$$\frac{\text{Magnetic dissipation terms}}{\text{External coupling terms}} \sim \frac{\lambda a^2/1^2}{Ha^2/1} = \frac{\lambda}{H1} = \frac{1}{R_{mH}}$$

where RH and RmH stand for the Reynolds number and the magnetic Reynolds number with respect to the mean Alfven speed H. If therefore, the imposed magnetic field is sufficiently strong 1/RH and 1/RmH are small in comparison with 1 and hence eqn. (3. 1) becomes

$$\frac{\partial}{\partial t}\overline{u_i' u_j''} + H_k\frac{\partial}{\partial \xi_k}\left(\overline{h_i' u_j''} - \overline{u_i' h_j''}\right)$$

$$= \frac{1}{\rho}\left(\frac{\partial \overline{p_*' u_j''}}{\partial \xi_i} - \frac{\partial \overline{u_i' p_*''}}{\partial \xi_i}\right)$$

$$+f\left(\overline{u_i' u_j''} - \overline{v_i' u_j''} + \overline{u_j'' v_i'} - \overline{v_j'' u_i'}\right)$$

$$-2\varepsilon_{nki}\Omega_n\overline{u_i' u_j''} - 2\varepsilon_{pkj}\Omega_p\overline{u_i' u_j''} \qquad (3.5)$$

Following M. OhJi (1964) we have

$$\left(\frac{\partial \overline{p_*' u_j''}}{\partial \xi_i} - \frac{\partial \overline{u_i' p_*''}}{\partial \xi_i}\right) = 0$$

$$\overline{v_i' u_j''} = \overline{u_j'' v_i'} = 0$$

Therefore, Eqn. (3.5) becomes

$$\frac{\partial}{\partial t}\overline{u_i' u_j''} + H_k\frac{\partial}{\partial \xi_k}\left(\overline{h_i' u_j''} - \overline{u_i' h_j''}\right)$$

$$= 2f\,\overline{u_i' u_j'} - 2(\in_{nki}\Omega_n + \in_{pkj}\Omega_p)\,\overline{u_i' u_j'} \qquad (3.6)$$

Similarly, equations (3.2), (3.3) and (3.4) can be written as

$$\frac{\partial}{\partial t}\overline{h_i' h_j''} + H_k\frac{\partial}{\partial \xi_k}\left(\overline{u_i' h_j''} - \overline{h_i' u_j''}\right) = 0 \qquad (3.7)$$

$$\frac{\partial}{\partial t}\overline{u_i' h_j''} + H_k\frac{\partial}{\partial \xi_k}\left(\overline{h_i' h_j''} - \overline{u_i' u_j''}\right) = 2f\left(\overline{u_i' h_j''}\right) - 2\in_{nki}\Omega_n\,\overline{u_i' h_j''} \qquad (3.8)$$

and

$$\frac{\partial}{\partial t}\overline{h_i' u_j''} + H_k\frac{\partial}{\partial \xi_k}\left(\overline{u_i' u_j''} - \overline{h_i' h_j''}\right) = f\left(\overline{h_i' u_j''}\right) - 2\in_{pkj}\Omega_p\,\overline{h_i' u_j''} \qquad (3.9)$$

The Fourier transform of various correlation tensors appearing in equations (3.6), (3.7), (3.8) and (3.9) are expressed as spectral tensors in the following form.

$$\overline{u_i' u_k''} = \int \phi_{ij}(\tilde{k},t)\,e^{ikr}\,dk$$

$$\overline{h_i' h_k''} = \int \psi_{ij}(\tilde{k},t)\,e^{ikr}\,dk$$

$$\overline{h_i' h_k''} = \int \Gamma_{ij}(\tilde{k},t)\,e^{ikr}\,dk$$

$$\overline{h_i' h_k''} = \int \tilde{\Gamma}_{ij}(\tilde{k},t)\,e^{ikr}\,dk$$

It should be noted that ϕ_{ij} and ψ_{ij} are true tensors but Γ_{ij} and $\tilde{\Gamma}_{ij}$ are skew tensors and that

$$\phi_{ij}(\tilde{k}) = \phi_{ji}(-\tilde{k}),\ \psi_{ij}(\tilde{k}) = \psi_{ji}(-\tilde{k}),\ \Gamma_{ij}(\tilde{k}) = \tilde{\Gamma}_{ji}(-\tilde{k}) \qquad (3.10)$$

From homogeneity

$$\left.\begin{aligned} k_i\phi_{ij}(\tilde{k}) = k_i\psi_{ij}(\tilde{k}) = k_i\Gamma_{ij}(\tilde{k}) = k_i\,\tilde{\Gamma}_{ij}(\tilde{k}) = 0 \\ k_i\phi_{ij}(\tilde{k}) = k_j\psi_{ij}(\tilde{k}) = k_j\Gamma_{ij}(\tilde{k}) = k_j\,\tilde{\Gamma}_{ij}(\tilde{k}) = 0 \end{aligned}\right] \qquad (3.11)$$

The spectral equations in the present context then becomes

$$\frac{d}{dt}\phi_{ij}(\tilde{k},t) - ik\mu H[\tilde{r}_{ij}(\tilde{k},t) - r_{ij}(\tilde{k},t)] = f\, r_{ij}(\tilde{k},t) - 2\in_{nki}\Omega_n + \in_{pkj}\Omega_p)\,\phi_{ij}(k) \quad (3.12)$$

$$\frac{d}{dt}\psi_{ij}(\tilde{k},t) - ik\mu H[\tilde{r}_{ij}(\tilde{k},t) - r_{ij}(\tilde{k},t)] = 0 \quad (3.13)$$

$$\frac{d}{dt}r_{ij}(\tilde{k},t) - ik\mu H[\phi_{ij}(\tilde{k},t) - \psi_{ij}(\tilde{k},t)] = f\, r_{ij}(\tilde{k},t) - 2\in_{nki}\Omega_n r_{ij}(\tilde{k},t) \quad (3.14)$$

and

$$\frac{d}{dt}\tilde{r}_{ij}(\tilde{k},t) - ik\mu H[\phi_{ij}(\tilde{k},t) - \psi_{ij}(\tilde{k},t)] = f\, \tilde{r}_{ij}(\tilde{k},t) - 2\in_{pkj}\Omega_n\, \tilde{r}_{ij}(k,t) \quad (3.15)$$

where μ denotes the cosine of the angle between k and H, i.e. $k\mu\, H = K_k.H_k$.

A remarkable feature of equations (3.12), (3.13), (3.14) and (3.15) is that their solutions are of oscillatory nature, such oscillations are caused and maintained by the imposed magnetic field which play an analogous role to the primary field of a conventional electric dynamo or motor.

For axis symmetry turbulence, we can put

$$\begin{bmatrix}\phi_{ij}(\tilde{k},t)\\ \psi_{ij}(\tilde{k},t)\\ r_{ij}(\tilde{k},t)\\ \tilde{r}_{ij}(\tilde{k},t)\end{bmatrix} = \begin{bmatrix}\phi^{(1)}\\ \psi^{(1)}\\ \gamma^{(1)}\\ \tilde{\gamma}^{(1)}\end{bmatrix}\Delta_{ij}(\tilde{k}) + \begin{bmatrix}\phi^{(2)}\\ \psi^{(2)}\\ \gamma^{(2)}\\ \tilde{\gamma}^{(2)}\end{bmatrix}\theta_{ij}(S.\tilde{k})$$

where S is a unit vector in the direction of H and

$$\left.\begin{aligned}&\Delta_{ij}(\tilde{k}) = (k^2\delta_{ij} - k_i k_j)/(k^2; \delta_{ij} \text{ is Kronecker's symbol}\\ &\theta_{ij}(s,\tilde{k}) = [k^2(1-\mu^2)\delta_{ij} - k_i k_j - k^2 S_i S_j + k\mu(S_i k_j + k_i S_j)]/k^2\end{aligned}\right] \quad (3.16)$$

While the defining scalars $\phi^{(2)}$ ……, , $\gamma^{(2)}$ are functions of k and $k\mu$ as well as time t, it follows from the homogeneity conditions that

$$\phi^{(1,2)}(k,k\mu) = \phi^{(1,2)}(k,-k\mu)$$

$$\psi^{(1,2)}(k,k\mu) = \psi^{(1,2)}(k,-k\mu)$$

for true tensors, and

$$\gamma^{(1,2)}(k,k\mu) = -\gamma^{(1,2)}(k,-k\mu)$$

for skew tensors, under these conditions the solutions are easily transformed in to a scalar form. The result is simply to replace ϕ_{ij} (k,t) by the corresponding scalars $\phi^{(1,2)}(k, k\mu, t)$ and so on.

Equations (3.12), (3. 13), (3. 14) and (3. 15) in this case are written as

$$\frac{d\phi^{(1)}}{dt} = 2f\phi^{(1)} - ik\mu H(\tilde{\gamma}^{(1)} - \gamma^{(1)}) - 2(\in_{nki}\Omega_n) + 2\in_{pkj}\Omega_k)\phi^{(1)}$$

$$\frac{d\phi^{(2)}}{dt} = 2f\phi^{(2)} - ik\mu H(\tilde{\gamma}^{(2)} - \gamma^{(2)}) - 2(\in_{nki}\Omega_n) + 2\in_{pkj}\Omega_k)\phi^{(2)}$$

$$\frac{d\psi^{(1)}}{dt} = ik\mu H(\tilde{\gamma}^{(1)} - \gamma^{(1)})$$

$$\frac{d\psi^{(2)}}{dt} = ik\mu H(\tilde{\gamma}^{(2)} - \gamma^{(1)})$$

$$\frac{d\gamma^{(1)}}{dt} = f\gamma^{(1)} + ik\mu H(\phi^{(1)} - \psi^{(1)}) - 2\in_{nki}\Omega_n\gamma^{(1)}$$

$$\frac{d\gamma^{(2)}}{dt} = f\gamma^{(2)} + ik\mu H(\phi^{(2)} - \psi^{(2)}) - 2\in_{nki}\Omega_n\gamma^{(2)}$$

$$\frac{d\gamma^{-(1)}}{dt} = f\gamma^{-(1)} + ik\mu H(\phi^{(1)} - \psi^{(1)}) - 2\in_{pki}\Omega_p\gamma^{-(1)}$$

$$\frac{d\gamma^{-(2)}}{dt} = f\gamma^{-(2)} + ik\mu H(\phi^{(2)} - \psi^{(2)}) - 2\in_{pki}\Omega_p\gamma^{-(2)}$$

where $\phi^{(1)}, \phi^{(2)}, \psi^{(1)}, \psi^{(2)}, \gamma^{(1)}, \gamma^{(2)}, \gamma^{-(1)}, \gamma^{-(2)}$ are defining scalars.

The effect of a strong external magnetic field on MHD turbulence has been discussed in a largely simplified form. The basic assumption is that if an imposed field is strong enough both the triple correlation terms and the dissipation terms will be of little significance. In other words, the reaction time of the external magnetic field has been assumed sufficiently small compared with the characteristic time scale of the decay process. When the external fields is steady and uniform, in particular, the solutions are very simple but appear to display some essential features of the magnetohydro-dynamic response of the turbulence. A point of future interest may perhaps be to elucidate the effect of the non steady or non uniform external fields as well as that of the non-linear energy-transfer mechanism.

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